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HORIZONTAL DISC RANDOM ARRAY PERFORMANCE

Hydrophone arrays of omnidirectional, cardioid, and iso-opt hypercardioid elements analyzed for noise gain and directional response in vertically directive noise

GE Martin

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METRIC CONVERSION

To convert from	To	Multiply by
feet	metres (m)	$\sim 3.05 \times 10^{-1}$
yards	m	$\sim 9.14 \times 10^{-1}$
nautical miles (nmi)	km	~ 1.85

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Acoustic arrays	Iso-opt hypercardioid elements	Beam widths
Horizontal disc random arrays	Performance (engineering)	Sidelobe levels
Omnidirectional elements	Ocean acoustic noise gain	Front-to-back response

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Noise gains and directional responses are studied for horizontal disc random arrays with omnidirectional or directional hydrophones. The directional elements are pressure-gradient hydrophones of hypercardioid and cardioid limaçon forms. Isotropic and vertically directive noise fields are considered along with a set of random arrays of the same relative shape. Since ocean noise below 200 Hz is due predominantly to shipping and above 200 Hz predominantly to wind, array sizes are studied that have design frequencies, f_D, of 25 to 300 Hz and mean interelement spacings of one wavelength at each design frequency. Performance is analyzed from 10 to 600 Hz. Above the design frequency, the well-known gain of $10 \log_{10}(N)$ is obtained with omnidirectional-

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elements; but over much of the decade below the design frequency, the gain falls off and is linearly related to frequency. The gain improvement due to directional elements – as little as 1 dB and as much as 6 dB – is especially significant at frequencies below 0.1 f_D and above f_D. The directional responses considered include beam widths (directional resolution), sidelobe levels, and front-to-back response improvement due to directional elements.

OBJECTIVES

Analyze the performance of horizontal disc random arrays of 16 omnidirectional, cardioid, or iso-opt hypercardioid elements in vertically directive and isotropic ambient sea noise. Assume that the arrays are formed by a (constrained) random selection of element locations but that they are operated over a wide frequency range for which the mean interelement spacing ranges from very small to very large (thinned). Analyze array noise gains and directional responses for these horizontal disc random arrays. Consider array sizes over a twelvefold range, all of the same shape. Determine the array performance at frequencies sufficiently low so that the mean interelement spacing is small and the gain is significantly less than $10 \log_{10}(N)$.

RESULTS

1. The use of limaçon directional hydrophones in arrays operated at frequencies for which the mean interelement spacing exceeds one wavelength or is less than 0.1 λ yields 3–6 dB greater array gain than is provided by arrays with omnidirectional elements.

2. Arrays with directional elements have nearly maximum gain with average interelement spacings of about one wavelength if the design frequency (f_D) is 300 Hz, but five-wavelength spacing is required if f_D is 25 Hz.

3. Compared to a single directional element alone, an array of 16 iso-opt hypercardioid elements provides a few dB less improvement in gain than an array of omnidirectional elements.

4. The horizontal disc array has nearly constant gain with azimuth even with substantial asymmetry of element locations.

5. Beam widths of random disc arrays in both horizontal and vertical planes approximate those of a solid horizontal disc. The vertical beam width is much greater than the horizontal beam width for the edge-fire disc. Large disc arrays such as those with a design frequency of 50 Hz have a significant negative signal gain for signals arriving at angles typical of many realistic environmental conditions. Their gain can be boosted by the use of vertical steering.

6. The directivity patterns have relatively high average sidelobe levels – about –12 dB – but this is to be expected for random arrays with 16 elements.

RECOMMENDATIONS

1. Consider the horizontal disc array, including the random form discussed, for possible applications.

2. Investigate lobe suppression with regular finite disc arrays.

3. Investigate ocean engineering of both regular and random discs.

CONTENTS

BACKGROUND

In the study of passive sonar system performance and the potential extension thereof, it was desired to determine how much improvement could be achieved by the use of horizontal arrays. The baseline system is a single element containing two hydrophones – one omnidirectional, the other a directional receiver. Both types of hydrophones are candidate elements for the arrays. NOSC TN 728 (ref 1) reported the noise gains and directional responses of single optimal iso-opt hypercardioid, cardioid, and omnidirectional hydrophones in vertically directive noise (ref 2) and in isotropic noise. A recent paper (ref 3) describes small directional hydrophones in a brief nonmathematical style.

This paper provides performance analyses of horizontal disc random arrays of 16 such elements. An extensive review of array technology, including much random array theory, was published by Steinberg (ref 4). He indicates that random arrays are perforce thinned* and provides the theory, a historical account, and references pertaining to the development thereof. Of course, no random array is thinned at arbitrarily low frequencies; but the theory can be tractably generalized at those higher frequencies for which the thinned condition prevails. Even underwater sound applications have been devoted almost exclusively to thinned random arrays. In this report the arrays are formed by a (constrained) random selection of element locations but are operated over a wide frequency range for which the mean interelement spacing ranges from very small (not thinned) to very large (thinned). This is one of the ways in which this work is new. It provides analyses of array noise gains and directional responses for these horizontal disc random arrays. Arrays with sizes over a twelvefold range, all of the same shape, are considered. One objective is to determine the array performance at frequencies low enough that the mean interelement spacing is small and the gain is significantly less than $10 \log_{10}(N)$, where N is the number of elements.

Prior work of the author considered the performance (especially array gain) of one-, two-, and three-dimensional arrays, including the effects of directional hydrophones (ref 5–7).

1. NOSC TN 728, Optimal Gains for Horizontal Directional Hydrophones in a Vertically Directive Noise Field, by GE Martin, 23 July 1979. NOSC TNs are informal documents intended chiefly for internal use.
2. MC Report 011, Acoustic Environmental Scenarios and Predictions for ASW, October 1972, vol 2-15, Long Range Acoustic Propagation Project, Ocean Science Program, Maury Center for Ocean Science, Department of the Navy.
3. Small-Aperture Directional Hydrophones, by GD Robertson; IEEE Publication 78 CH 1354-4 AES, EASCON '78 Record, IEEE, 25–27 September 1978, p 278–281.
4. Principles of Aperture and Array System Design, by BD Steinberg, John Wiley & Sons, NY, 1976, p 139–189.
5. Array Gain Variations Due to Noise Anisotropy, by GE Martin; IEEE Publication 75 CHO 998-5, IEEE EASCON 1975 Record, September–October 1975, p 67A–67J.
6. Gain of Cylindrical Arrays in Anisotropic Sea Noise, by GE Martin, paper L3, Ninth International Congress on Acoustics, Madrid, July 1977.
7. Underwater Array Design as Influenced by Background Noise and Propagation Effects, by GE Martin, Proceedings of Conference on Sound Propagation and Underwater Systems, (British) Institute of Acoustics, Underwater Acoustics Group, Imperial College, London, 10 April 1978.

*Steinberg (ref 4, p 139) severely limits random arrays as follows: "The random array is one form of aperiodic array. It is a thinned array (mean interelement spacing greater than λ/2) and therefore is less costly in components than a conventional phased array."

Those studies used vertically directive noise (ref 2), but for a different set of ocean environmental parameters. For well-sampled horizontally planar arrays, greater improvement of gain was achieved with the plane's elements configured as upward-null cardioid hydrophones than for relatively thin ($T < 2\lambda$) volumetric configurations (ref 5). It was also demonstrated that array gain varies significantly with the noise model; thus the best available model should be used for system analyses. This is generally not critical for arrays that are undersampled, such as large random arrays that contain relatively few elements. Although that prior work provides useful general information, specific results are needed for random arrays with several types of hydrophone directionalities.

Both in this report and the prior work, we neglect all noise other than ambient sea noise. The noise of the hydrophone due to flow and (vertical) motion can be significant - especially so for gradient-type hydrophones (ref 8).

COORDINATE SYSTEM

Figure 1 shows the usual spherical coordinate system (ref 9) that we will use, where

$x = R \sin \theta \cos \phi$

$y = R \sin \theta \sin \phi$

$z = R \cos \theta .$

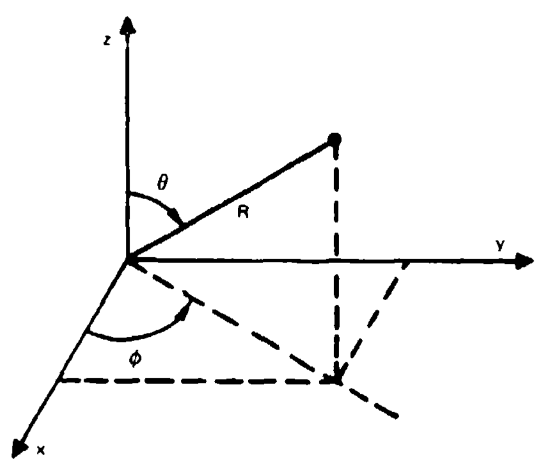

Figure 1. Spherical coordinate system.

GENERAL CHARACTER OF SEA NOISE

The environmental noise levels typical of ocean areas (ref 10) are shown in figure 2 for the low- to medium-frequency region. The low frequencies are dominated by shipping

8. Measurements of Low-Velocity Flow Noise on Pressure and Pressure Gradient Hydrophones, by RA Finger, LA Abbagnaro, and BB Bauer; J Acoust Soc Am, vol 65, June 1979, p 1407-1412.
9. Methods of Theoretical Physics, by PM Morse and H Feshbach, vol I, p 658, McGraw-Hill Book Co, NY 1953.
10. Mechanics of Underwater Noise, by D Ross; Pergamon Press, NY, 1976, p 71, 281.

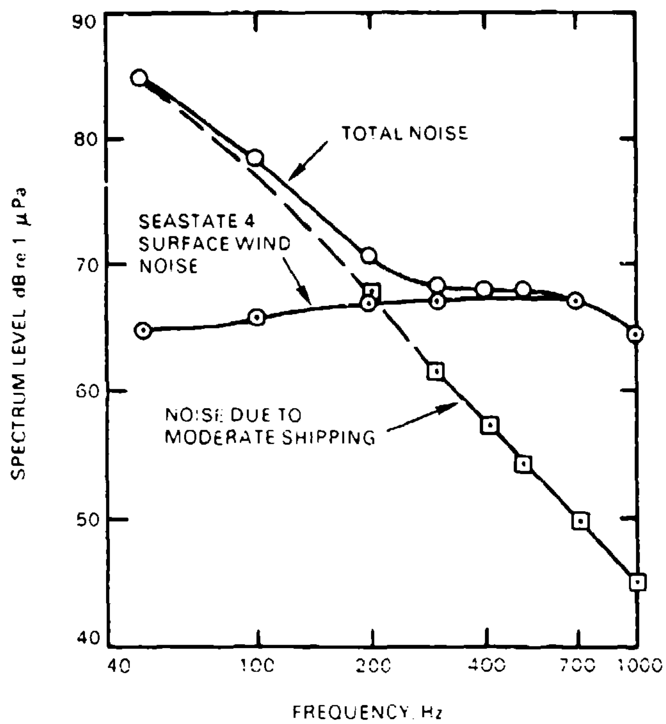

Figure 2. Typical ambient sea noise.

noise from discrete azimuths along certain vertical arrival angles, from both open-ocean and bottom-coupled contributions. The middle frequencies are dominated by surface wind noise that is predominantly uniform in azimuthal distribution. Thus, realistic representations of ambient noise must include horizontal directionality. The discussion here is limited to noise with realistic vertical directionality but omnidirectional character in all horizontal planes.

The frequency dependence of noise evident in figure 2 reveals a change in the dominant noise mechanism above about 200 Hz; the actual crossover frequency depends somewhat upon the relative levels due to shipping and surface wind. System performance generally is better above this frequency because the noise levels are lower. Since noise is an important factor in the consideration of system performance, it was considered vital in this generalized study to use a wide range of frequencies — up to 600 Hz.

The character of the noise model is that of a time-invariant ocean in which ships radiate noise uniformly. There are no nearby ships that otherwise would dominate the noise field.

AMBIENT SEA NOISE

This study relates primarily to directional ambient sea noise that has vertical (θ) dependence but no horizontal (φ) dependence. The sea noise intensity per unit solid angle (steradian) is considered to be uniform in all horizontal planes and is given by the relationship

$\tilde{n}(\theta, \phi) = n(\theta)$.

Such noise field data are used for passive sonar system performance studies (ref 1, 4, 5, 6) and other undersea environmental resources such as the Long Range Acoustic Propagation Project (LRAPP) (ref 2). Also of this vertically directive nature are some wind noise forms such as the idealized surface dipole model.

The entire study was devoted to one ocean area. The sound speed profile is shown in figure 3. The water has a depth of 3000 metres (9843 feet). This study was limited to a source depth of 60 feet and a receiver depth of 9793 feet, 50 feet above the bottom. The noise data for these conditions are listed in table 1 and are graphically displayed in figure 4.

DIRECTIONAL HYDROPHONE ARRAY-ELEMENT RESPONSES

Since the various candidate hydrophone elements are discussed in detail in ref 1, a mere brief summary is appropriate here. The omnidirectional and gradient hydrophones all possess a directional response of the limaçon form of Pascal:

$$p = \frac{A}{A + B} + \frac{B}{A + B} \cos(\phi - \phi_H) \sin(\theta),$$

where ϕ_H is the effective steering angle of the hydrophone in the horizontal plane and

$\theta_H = \frac{\pi}{2}$.

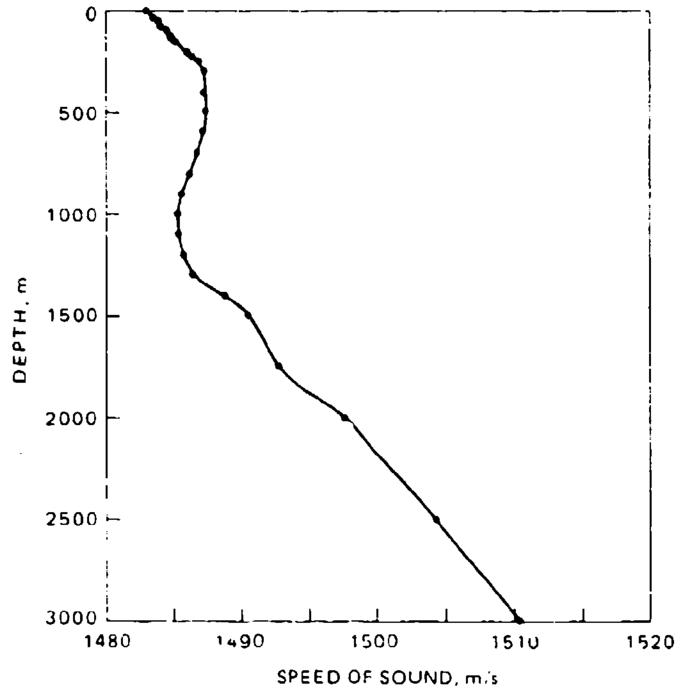

Figure 3. Sound speed profile for an ocean area.

Source Depth = 60 feet (18.3 m)
Receiver Depth = 9793 feet (2985 m)

θ_{min}	θ_{max}	Frequency, Hz: 10	50	100	300	900	2400	Isotropic Noise Level
0.0	25.8	48.4	48.4	47.5	45.0	42.3	38.1	57.0
25.8	36.9	48.2	48.2	47.1	44.5	41.8	37.9	57.0
36.9	45.6	49.1	49.1	47.7	44.0	41.3	37.6	57.0
45.6	53.1	50.3	50.3	48.6	43.5	40.6	37.2	57.0
53.1	60.0	51.9	51.9	49.8	42.9	39.9	36.6	57.0
60.0	66.4	53.8	53.8	51.3	43.2	39.2	36.1	57.0
66.4	72.5	56.3	56.3	53.1	45.1	40.0	35.5	57.0
72.5	78.5	60.2	60.2	56.4	49.1	43.0	35.0	57.0
78.5	84.3	63.6	63.6	59.1	52.5	45.2	34.4	57.0
84.3	90.0	73.0	73.0	67.3	57.7	46.2	36.3	57.0
90.0	95.7	73.3	73.3	67.7	58.7	49.6	42.0	57.0
95.7	101.5	65.6	65.6	61.1	55.9	50.6	44.0	57.0
101.5	107.5	63.3	63.3	59.5	55.4	51.4	45.7	57.0
107.5	113.6	61.5	61.5	58.3	55.6	52.2	47.1	57.0
113.6	120.0	60.7	60.7	58.1	56.2	53.1	48.2	57.0
120.0	126.9	60.3	60.3	58.2	56.9	53.9	49.1	57.0
126.9	134.4	60.1	60.1	58.5	57.5	54.6	49.9	57.0
134.4	143.1	60.1	60.1	58.8	58.0	55.2	50.6	57.0
143.1	154.2	60.2	60.2	59.1	58.5	55.8	51.2	57.0
154.2	180.0	60.4	60.4	59.5	59.0	56.2	51.7	57.0
Omni Level		75.7	75.7	71.1	66.1	62.1	57.1	70.0

Table 1. Vertical noise directionality in dB re 1 W/sr.

Some characteristics of several hydrophones are presented in table 2. The directional hydrophones are used to achieve higher gain. A single iso-opt* unit has a gain of 4.9 to 6.9 dB in this vertically directive noise.

The directional responses of iso-opt hypercardioid, cardioid, and (one kind of) optimal hydrophones are shown in figure 5. The optimal unit in isotropic noise is called the iso-opt hypercardioid here to differentiate it from other limaçons. It has nearly optimal gain for this vertically directive noise, as shown in table 2. This report assesses the gain improvement of arrays using such directional elements.

*An iso-opt hydrophone is a hypercardioid limaçon with parameters A = 0.25 and B = 0.75, which are optimum for gain in isotropic noise.

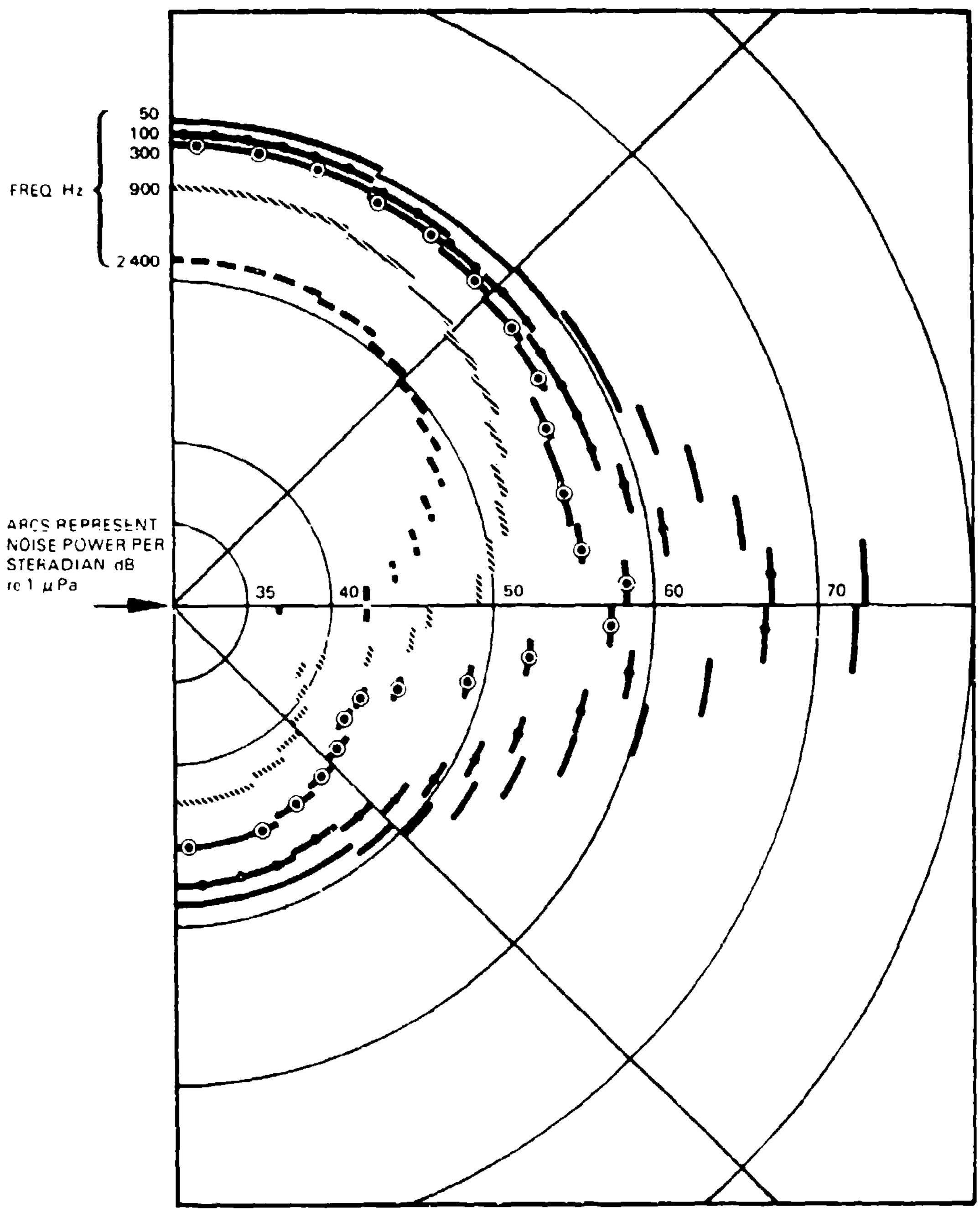

Figure 4. Vertical noise directionality for an ocean area.

Type		Omnidirectional	Cardioid	Iso-opt	Optimal
A		1.000	0.500	0.250	variable
B		0.000	0.500	0.750	variable
Gain in Isotropic Noise, dB		0.000	4.771	6.021	6.021
Gain in Vertically Directive Noise, dB	f = 10	0.000	4.366	4.904	4.993
	f = 50	0.000	4.366	4.904	4.993
	f = 100	0.000	4.472	5.179	5.230
	f = 300	0.000	4.790	6.078	6.079
	f = 900	0.000	4.969	6.642	6.674
	f = 2400	0.000	5.038	6.875	6.936

Table 2. Some characteristics of hydrophones.

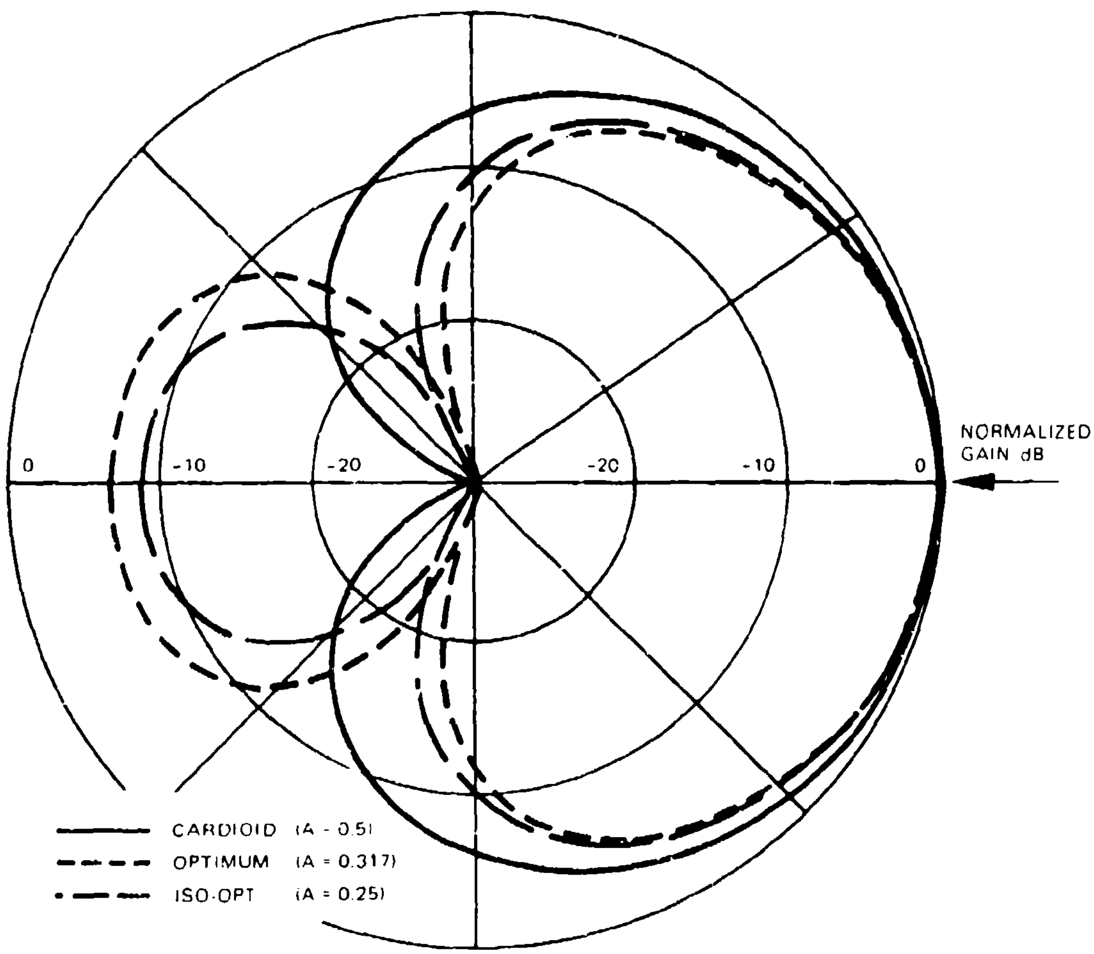

Figure 5. Directivity patterns for three hydrophones.

ARRAY DESIGN

Consider a set of arrays of various sizes but of proportional dimensions. It is useful to define the design frequency, f_D, such that the mean interelement spacing is one wavelength:

$$\pi a^2 = N(\lambda_D)^2 = N(c/f_D)^2,$$

where

N = number of array elements

a = radius of the disc

c = speed of sound.

If we assume

N = 16 and c = 5000 feet per second,

then

$$a = \frac{11\,284}{f_D} .$$

The element locations were determined on the basis of such a radius. Let u(t) be a random variable with values uniformly distributed over the range

$$0 \leq u(t) \leq 1$$

for a domain of t of all positive integers. Then let

$$x(\ell) = 2u(t) - a$$

$$y(\ell) = 2u(t + 1) - a,$$

such that

$$[r(\ell)]^2 = [x(\ell)]^2 + [y(\ell)]^2 \leq (a)^2.$$

That is, we reject all output pairs that would produce (x,y) pairs outside a circle of radius a. Our first 56 such valid locations are shown in figure 6. The first 16 locations are noted with a circle; the second 16 with the symbol X. For a thorough study many sets of locations would be analyzed, but neither funding nor time would permit that. Instead, a single realistic distribution was used. The first 16 locations as a [illegible] were rejected. Among other reasons were (1) the nearly identical coordinates for elements 11 and 14 and (2) the small effective radius. The second 16 locations were used for all random arrays and are shown for unit radius in figure 7. Arrays studied were scaled for these reference dimensions to produce arrays with design frequencies, radii, and diameters as shown in table 3. The actual coordinates for the array with f_D = 50 Hz are given in table 4. Only the three arrays with design frequencies of 50, 150, and 300 Hz were studied completely, sonar system analyses included. All were investigated for the noise gain and directional characteristics information reported here.

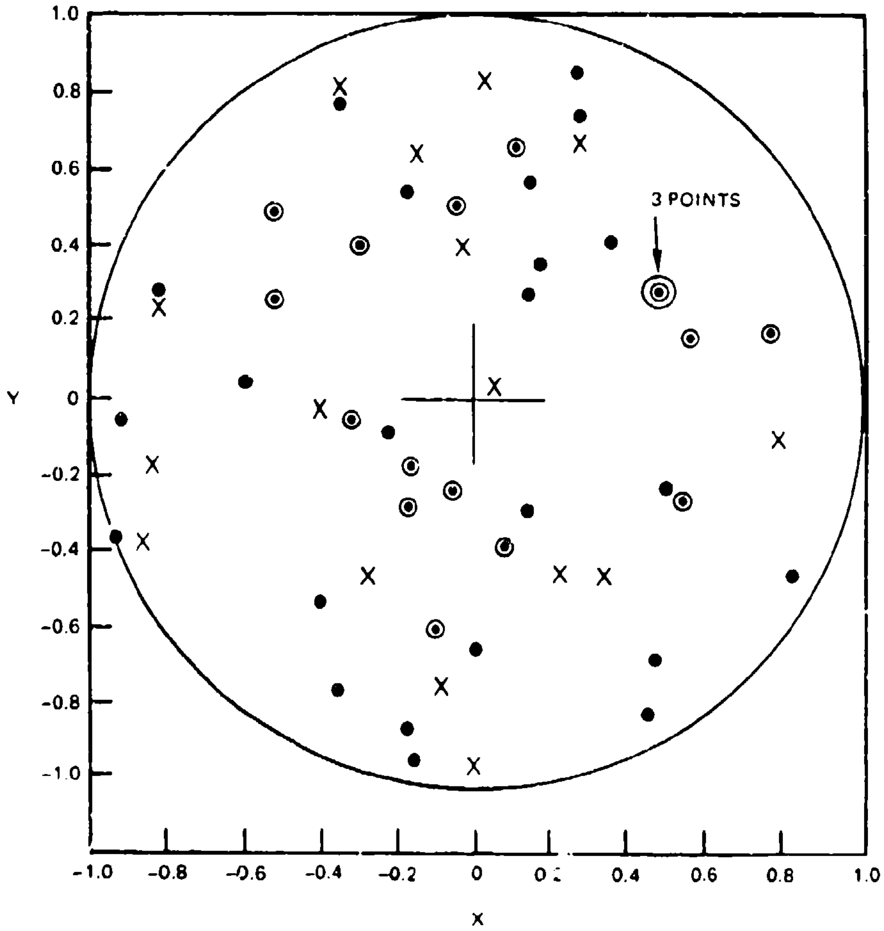

Figure 6. The first 56 randomly-selected locations for horizontal disc arrays.

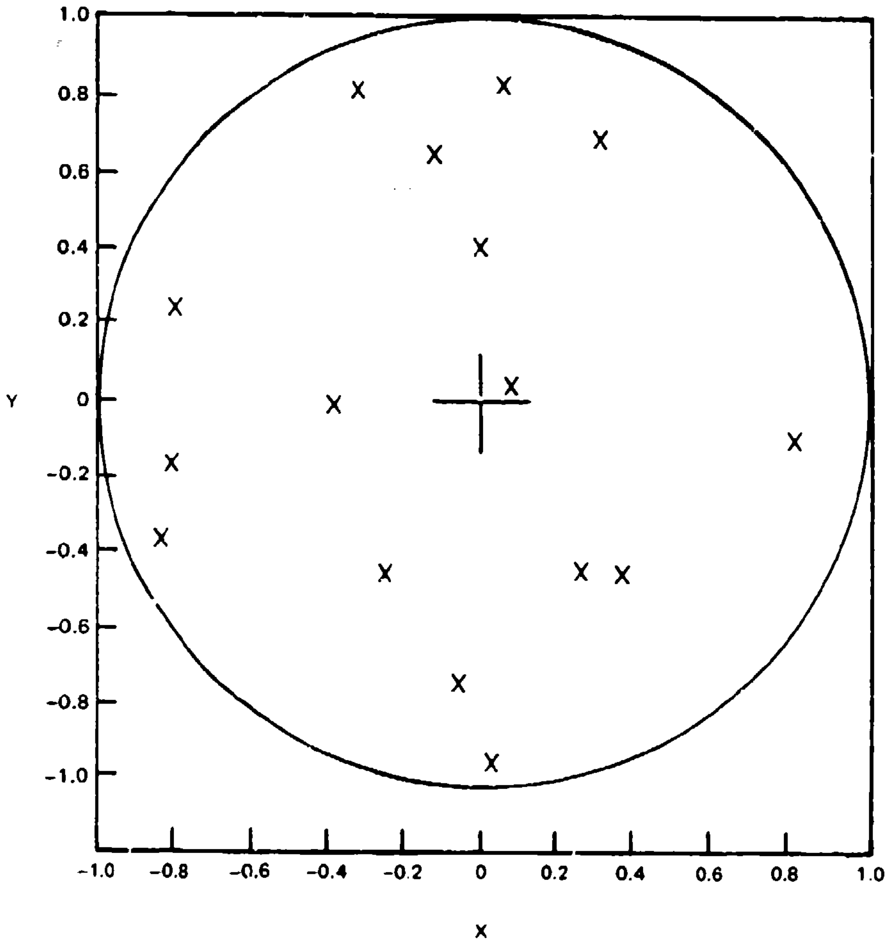

Figure 7. Locations of 16-element arrays studied, scaled for unit radius.

f_D, Hz	25	50	75	100	125	150	225	300
a, feet	450	225	150	112.5	90	75	50	37.5
Dia, feet	900	450	300	225	180	150	100	75

Table 3. Design frequencies, radii, and diameters of arrays.

ℓ	x(ℓ)	y(ℓ)	z(ℓ)	r(ℓ)	a
1	129.86	36.49	.00	134.89	225.00
2	-24.40	-134.60	.00	136.79	225.00
3	-38.41	-42.62	.00	57.37	225.00
4	-116.52	108.09	.00	158.94	225.00
5	-13.08	113.18	.00	113.94	225.00
6	-70.68	-9.56	.00	71.32	225.00
7	176.86	42.37	.00	181.87	225.00
8	-120.50	56.45	.00	133.07	225.00
9	-71.07	90.15	.00	114.79	225.00
10	120.81	-60.89	.00	135.28	225.00
11	114.50	65.47	.00	131.90	225.00
12	-42.37	-65.83	.00	78.29	225.00
13	25.82	150.76	.00	152.96	225.00
14	114.72	60.89	.00	129.88	225.00
15	19.99	-89.78	.00	91.98	225.00
16	-12.72	-55.61	.00	57.04	225.00
17	-187.75	-40.93	.00	192.16	225.00
18	17.84	12.56	.00	21.82	225.00
19	-10.23	92.68	.00	93.24	225.00
20	.16	-212.07	.00	212.07	225.00
21	79.94	-105.62	.00	132.46	225.00
22	-63.03	-99.09	.00	117.44	225.00
23	-183.09	53.18	.00	190.66	225.00
24	-34.58	146.84	.00	150.86	225.00
25	178.15	-22.45	.00	179.56	225.00
26	-21.12	-167.28	.00	168.60	225.00
27	58.20	-101.50	.00	117.00	225.00
28	2.99	184.70	.00	184.73	225.00
29	-90.22	-3.56	.00	90.29	225.00
30	-199.63	-85.20	.00	217.05	225.00
31	-79.58	185.44	.00	201.79	225.00
32	65.57	154.01	.00	167.39	225.00
33	31.48	66.18	.00	73.29	225.00
34	63.82	167.60	.00	179.33	225.00
35	-41.78	-196.09	.00	200.49	225.00
36	-39.65	120.77	.00	127.11	225.00
37	-134.13	13.84	.00	134.85	225.00
38	-79.59	178.78	.00	195.69	225.00
39	-80.35	-38.77	.00	89.22	225.00
40	109.22	56.65	.00	123.03	225.00
41	-207.97	-79.78	.00	222.75	225.00
42	34.06	-65.88	.00	74.16	225.00
43	-89.84	-119.60	.00	149.58	225.00
44	100.54	-186.21	.00	211.62	225.00
45	-49.16	-20.08	.00	53.10	225.00
46	63.83	193.05	.00	203.33	225.00
47	-82.17	-168.64	.00	187.59	225.00
48	-35.64	-214.96	.00	217.90	225.00
49	-.91	-150.26	.00	150.26	225.00
50	115.76	-55.18	.00	128.24	225.00
51	-211.00	16.51	.00	211.64	225.00
52	-16.95	-168.16	.00	169.01	225.00
53	43.57	86.46	.00	96.82	225.00
54	80.47	92.63	.00	122.71	225.00
55	-186.68	60.97	.00	196.31	225.00
56	191.06	-103.12	.00	217.12	225.00
57	109.12	-154.00	.00	188.74	225.00

Table 4. Coordinates for disc array with design frequency of 50 Hz.

ARRAY NOISE GAIN

Array gain is probably the most important array characteristic, since detectability is proportional to gain and higher gain also improves localization (resolution). Extensive calculations of array noise gain were made by using the NOSC PASS computer model (ref 11). Array signal gain was not considered here. Noise was assumed to be either vertically directive or isotropic. (For the latter, array gain is called directivity index.) Table 5 and appendix A give the gains for arrays in the vertically directive noise field. Tabulations are provided for arrays with iso-opt elements and those with omnidirectional elements. The arrays and the directional elements are steered for main-beam response at $\phi_S = 45°$ and $\theta_S = 90°$. Appendix A is a detailed listing of gains with iso-opt and omnidirectional elements as well as the difference of those two gains, to show the improvement of gain due to the use of directional elements.

The general character of array noise gain is presented in figure 8 as a function of the relative frequency, f/f_D. The data are displayed with separate symbols for each of the eight array sizes. Since the gains were computed at fixed frequencies and the design frequencies range from 25 to 300 Hz, the abscissa values exhibit a spread. Nonetheless, this type of presentation is very effective for the evaluation of general trends. The gains of arrays with omnidirectional hydrophones have relatively small scatter, approaching the $10 \log_{10}(N)$ value of 12 dB for frequencies above the design frequency in both vertically directive and isotropic noise. For arrays with iso-opt hypercardioid elements, gains scatter significantly for frequencies above 0.5 f_D for the vertically directive noise but the scatter is small for isotropic noise. For both noise fields, the high-frequency gains approach 18 dB (the sum of the gain due to an array with 16 independent omnidirectional elements (12 dB) and the gain of an iso-opt hypercardioid hydrophone). The smaller arrays, eg with f_D = 300 Hz, approach maximum gain at a lower relative frequency in this vertically directive noise field. The improvement due to the use of iso-opt hypercardioid elements varies from 1 to 6 dB. At relative frequencies below 0.09 f_D, the gain due to the simple hypercardioid is more than that due to the array. The effects due to the iso-opt are also significantly beneficial for frequencies above the design frequency.

These results lead to some important conclusions relative to interelement spacing in array design. In general it is desirable to space array elements so that the pair coherence in expected noise fields is relatively small. Such spacings provide nearly maximum gain for the array. If much smaller spacings are used, the array gain is smaller for the same system cost. To reduce cost, it is necessary to use the minimum number of elements and associated electronic channels. The necessary operation over a band of frequencies, however, entails some compromise. These relationships are summarized in table 6.

For omnidirectional hydrophones, the array gain of the disc is within 1 dB of its maximum (high-frequency) value for interelement spacings of 0.7 to 0.8 wavelength, for all noise directionalities considered. It is significant both that the spacing required is smaller than one wavelength and that the results are valid for widely different noise directionalities.

11. NOSC Technical Note NUC TN 1758, Performance Analysis for Surveillance System (PASS), User's Guide, by JL Hofmockel, JW Aitkenhead, and LK Arndt, September 1976.

f_D, Hz	25	50	75	100	125	150	225	300
Dia, feet	900	450	300	225	180	150	100	75

Iso-opt elements

Freq, Hz								
10	10.5	8.0	6.9	6.3	5.9	5.7	5.3	5.1
25	13.5	11.4	10.0	8.9	8.0	7.5	6.5	6.0
50	15.0	13.5	12.4	11.4	10.5	10.0	8.4	7.5
100	16.0	15.6	14.5	14.0	13.3	12.9	11.3	10.3
150	16.7	16.4	15.8	15.2	14.6	14.3	13.2	12.1
250	17.8	17.2	17.0	16.7	16.7	16.4	15.4	14.7
300	17.9	17.8	17.5	17.5	17.2	17.1	16.4	15.7
600	18.4	18.2	18.4	18.3	17.8	18.1	18.0	17.8

Omnidirectional elements

Freq, Hz								
10	8.7	6.5	5.0	3.5	2.4	1.8	0.8	0.5
25	11.2	8.9	8.7	7.7	6.5	5.9	4.1	2.6
50	11.6	11.2	10.3	8.9	8.7	8.7	7.0	5.9
100	11.6	11.7	10.7	11.3	10.6	10.4	8.7	8.8
150	11.7	11.6	11.8	11.7	10.9	11.3	10.5	9.3
250	12.1	11.8	11.7	11.5	11.9	11.6	11.3	11.0
300	11.9	11.9	11.8	11.9	11.7	11.9	11.5	11.5
600	12.1	12.0	12.2	12.0	11.8	11.8	12.1	12.0

Improvement in gain due to iso-opt elements

Freq, Hz								
10	1.8	1.5	1.9	2.8	3.5	3.9	4.5	4.7
25	2.3	2.5	1.3	1.2	1.5	1.6	2.5	3.4
50	3.5	2.3	2.2	2.5	1.8	1.3	1.4	1.6
100	4.4	3.8	3.8	2.7	2.8	2.5	2.5	1.5
150	5.0	4.8	4.1	3.6	3.7	3.0	2.7	2.9
250	5.7	5.4	5.3	5.2	4.8	4.8	4.1	3.7
300	5.9	5.9	5.8	5.7	5.6	5.2	5.0	4.3
600	6.3	6.3	6.1	6.3	6.1	6.3	5.9	5.8

Table 5. Array gains (dB) for horizontal disc random arrays in vertically directive noise ($N = 16$, $\phi_S = 45°$, $\theta_S = 90°$).

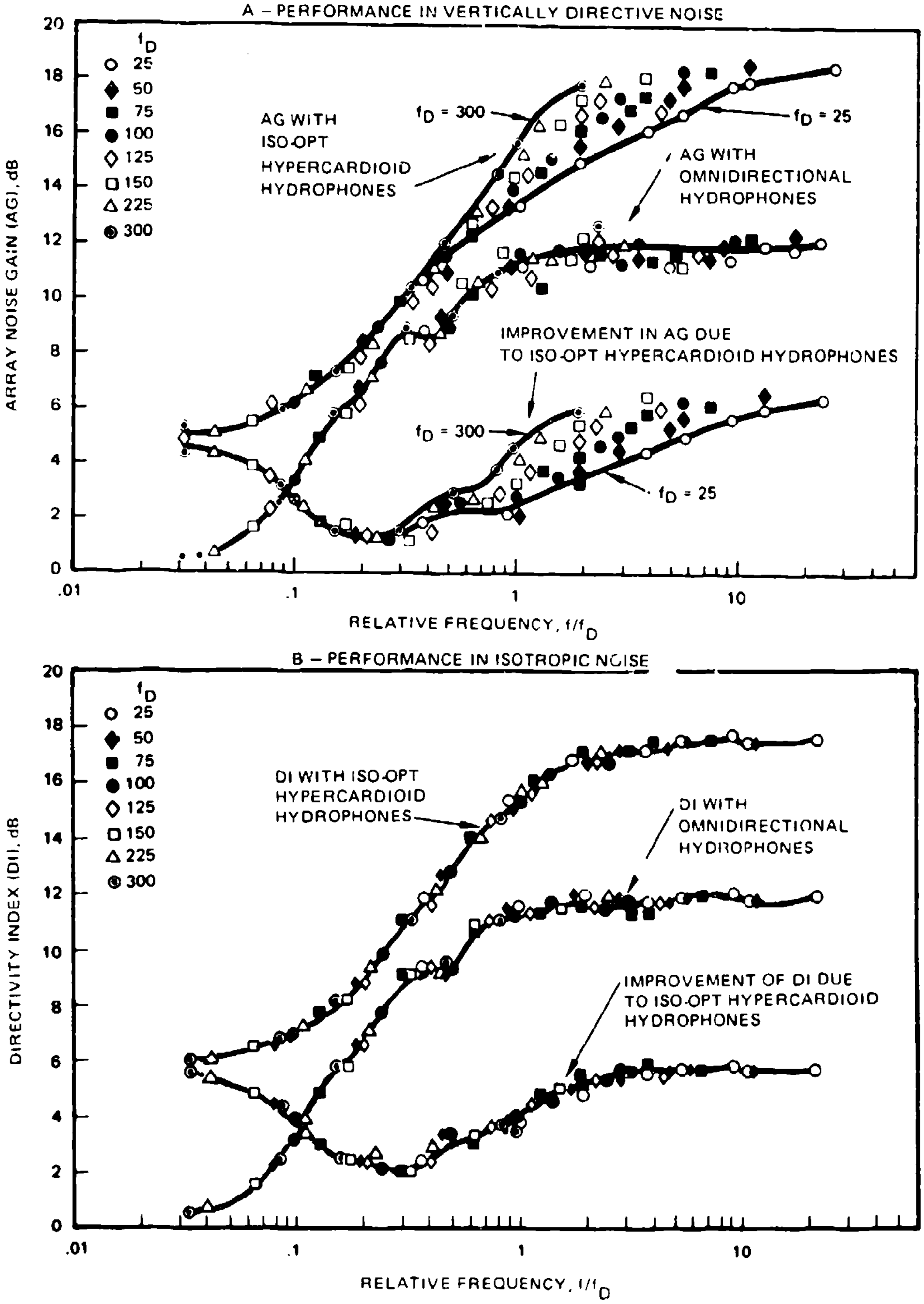

Figure 8. Noise gains for horizontal disc random arrays with omnidirectional and with iso-opt hypercardioid elements.

Hydrophone Type	Noise Directionality			
	Isotropic	Vertically Directive		
		$f_D = 300$	$f_D = 100$	$f_D = 25$
Omnidirectional	0.7	0.8	0.8	0.8
Iso-opt hypercardioid ($f_D \cdot d/\lambda$)	2.0	1.1 (330)	2.0 (200)	5.0 (125)

Table 6. Minimum relative interelement spacing, d/λ, to achieve array gain within 1 dB of maximum (disc arrays).

The results may be significantly different if iso-opt hypercardioid hydrophones are used, however. For isotropic noise, the minimum spacing is two wavelengths. For vertically directive noise, the minimum spacing varies from 1.1 λ for high-frequency arrays ($f_D = 300$) to 5 λ for low-frequency arrays ($f_D = 25$).

These are important observations. The higher-gain arrays using directional elements achieve maximum gain at wider spacings. This effect is substantial for low-frequency arrays. However, mid-frequency arrays ($f_D = 300$ Hz) require only a slightly larger spacing. The product $f_D \cdot d/\lambda$ given in table 6 indicates the frequency at which the array with directional hydrophones is within 1 dB of maximum gain. This is 330 Hz for an array with $f_D = 300$ Hz, 125 Hz for an array with $f_D = 25$ Hz.* It is readily apparent that arrays must be inordinately large to achieve high gain at low frequencies, much more than one might conclude from the linear relation between frequency and wave number ($c = f\lambda$).

The improvement of gain due to the use of iso-opt hypercardioid hydrophones is shown in figure 9 for a single array ($f_D = 150$ Hz). The gains with omnidirectional, cardioid, and iso-opt hypercardioid hydrophones are given. The difference between the array noise gains with iso-opt and with omnidirectional elements is due to the use of the directional hydrophones – an improvement that ranges from 1 to 6 dB. This paper does not consider the hydrophone self-noise issues, but the improvement derived by the use of directional hydrophones must offset the disadvantages such as increased self-noise and lower resultant array gain.

The gain improvement due to the difference of iso-opt and omnidirectional elements has been considered above. A primary issue in this study is the gain improvement due to the array. This is illustrated in figure 10 for the array with $f_D = 150$ Hz. The gains for an array of iso-opt hydrophones and for an array with omnidirectional elements are shown. Gains are also shown for a single iso-opt hydrophone alone (from data in table 2). The differences of the gains of an array of iso-opt elements and those of a single iso-opt hydrophone represent the improvement due to the array. Although the improvement is less than the array gain with omnidirectional hydrophones, it is significant: at $f = f_D = 150$ Hz, the improvement of 9 dB would triple the sonar range in a cylindrically spreading propagation environment. Even so, the use of an array of these hydrophones may not be as cost effective as a widely distributed field of single hydrophones.

*This is evident also when data are plotted as in the next few figures.

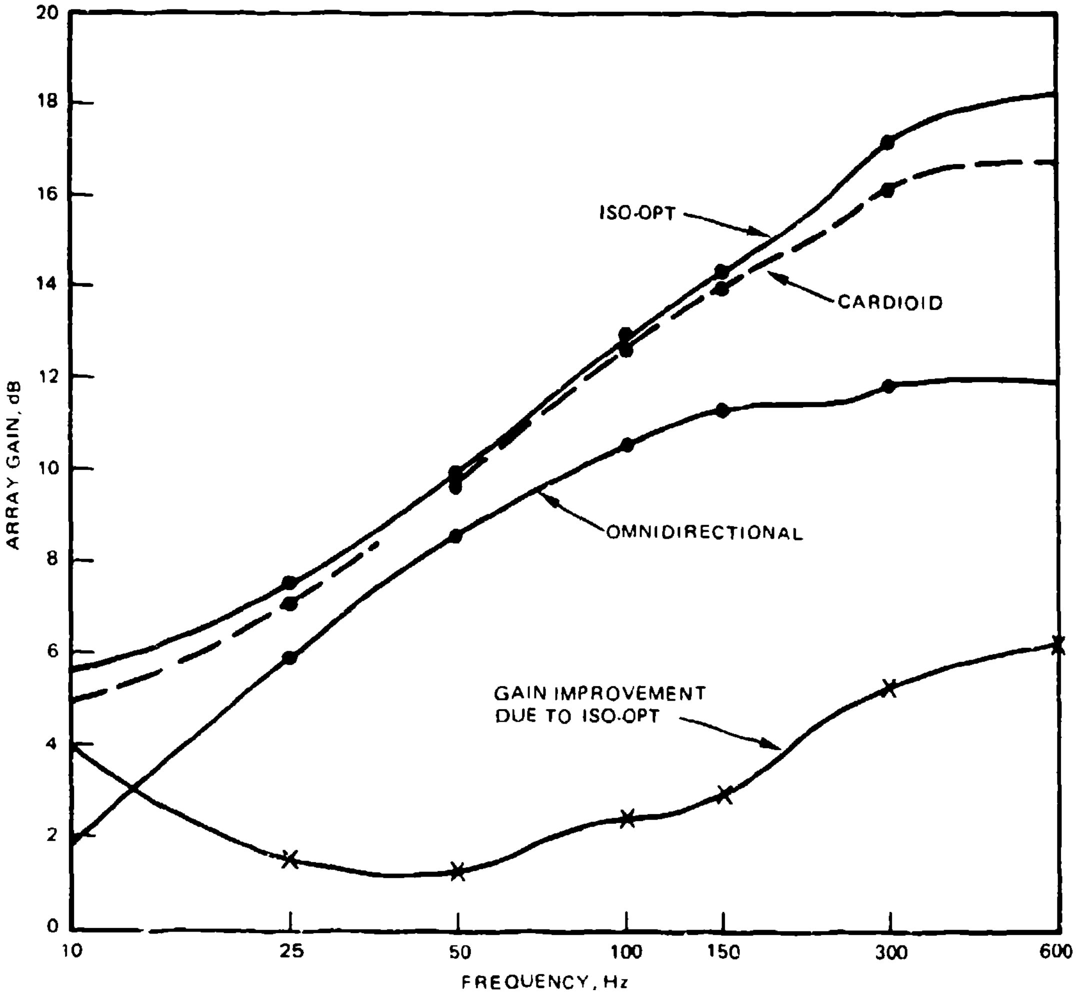

Figure 9. Gains for arrays with various types of hydrophones in vertically directive noise ($N = 16$, $f_D = 150$, $\phi_S = 45°$).

The effects of design frequency (ie array diameter) are shown in figure 11. The curves are very smooth for arrays with iso-opt elements, but they vary more with frequency if the elements are omnidirectional.

Almost all results described in this report relate to random disc arrays steered to $\theta_S = 90°$ and $\phi_S = 45°$, ie in the horizontal plane 45° from the x and y axes. Because the locations of array elements as shown in figure 7 were determined by a random number generator, the array is not symmetrical. The gains for three arrays with omnidirectional elements are shown in figure 12 for $\phi_S = 0°$, 45°, and 90°. Apparently a relatively small variation in array gain versus azimuth exists that is due to the substantial asymmetry of the array. It is important to recall that the noise model is nondirectional in horizontal planes. That is often not true in practice, and gain can change with azimuth. Disc arrays, however, have small gain changes with azimuth even if they are random arrays as illustrated here.

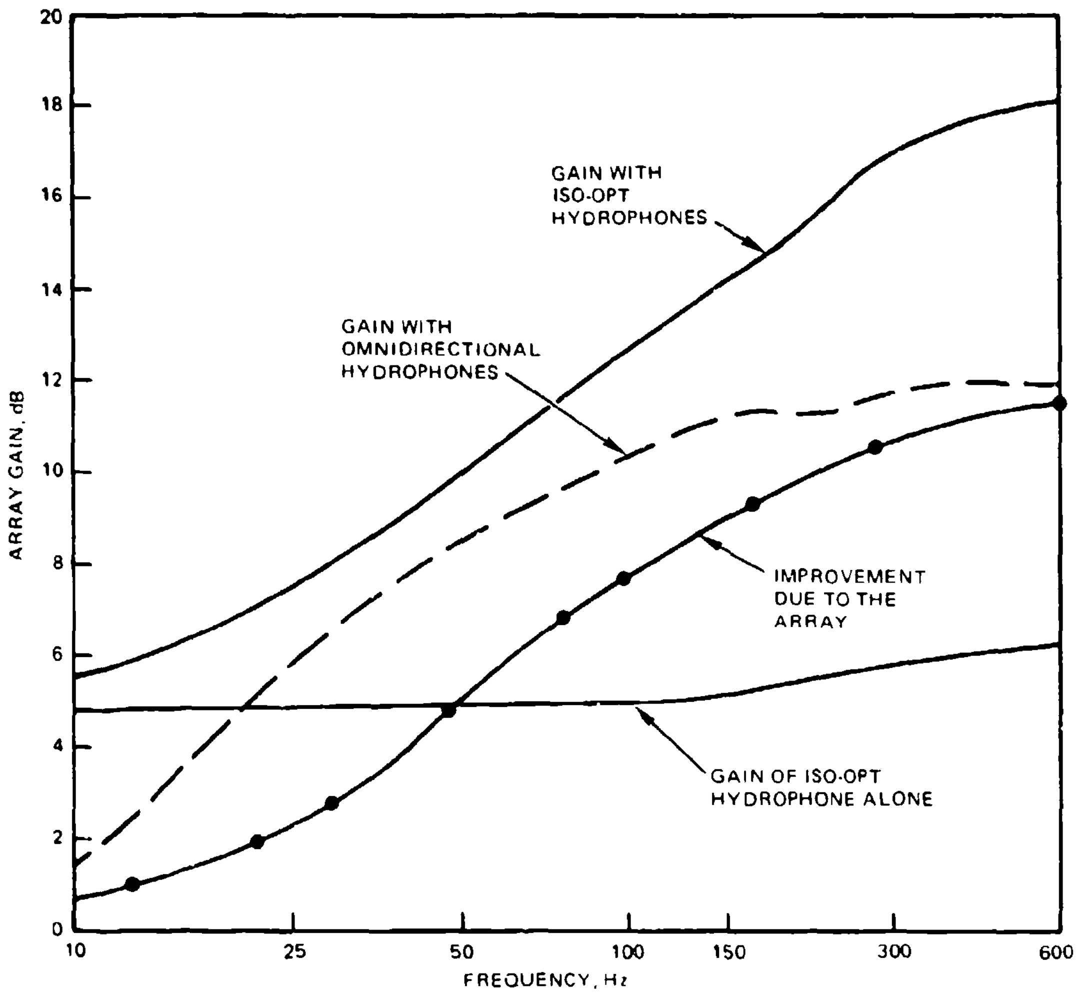

Figure 10. Improvement of gain due to the array in vertically directive noise ($N = 16$, $f_D = 150$, $\varphi_S = 45°$).

All of the prior results apply to the vertically directive noise field. The gains were also calculated for isotropic noise, and these are listed in table 7. Since the differences in gains for directional noise (see table 5) and isotropic noise are small, these data are presented for the record without illustrations.

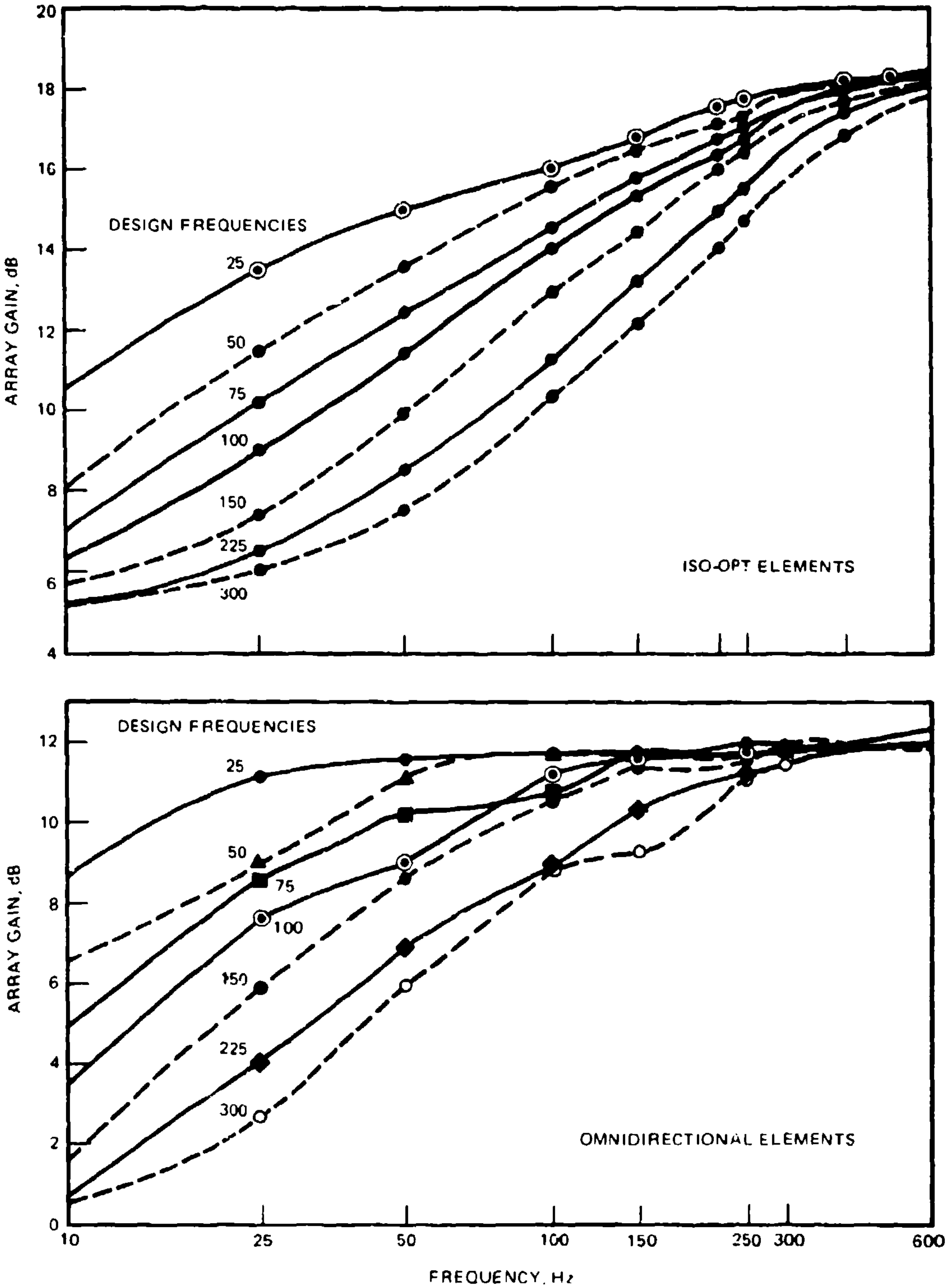

Figure 11. Effects of design frequency on gain in vertically directive noise.

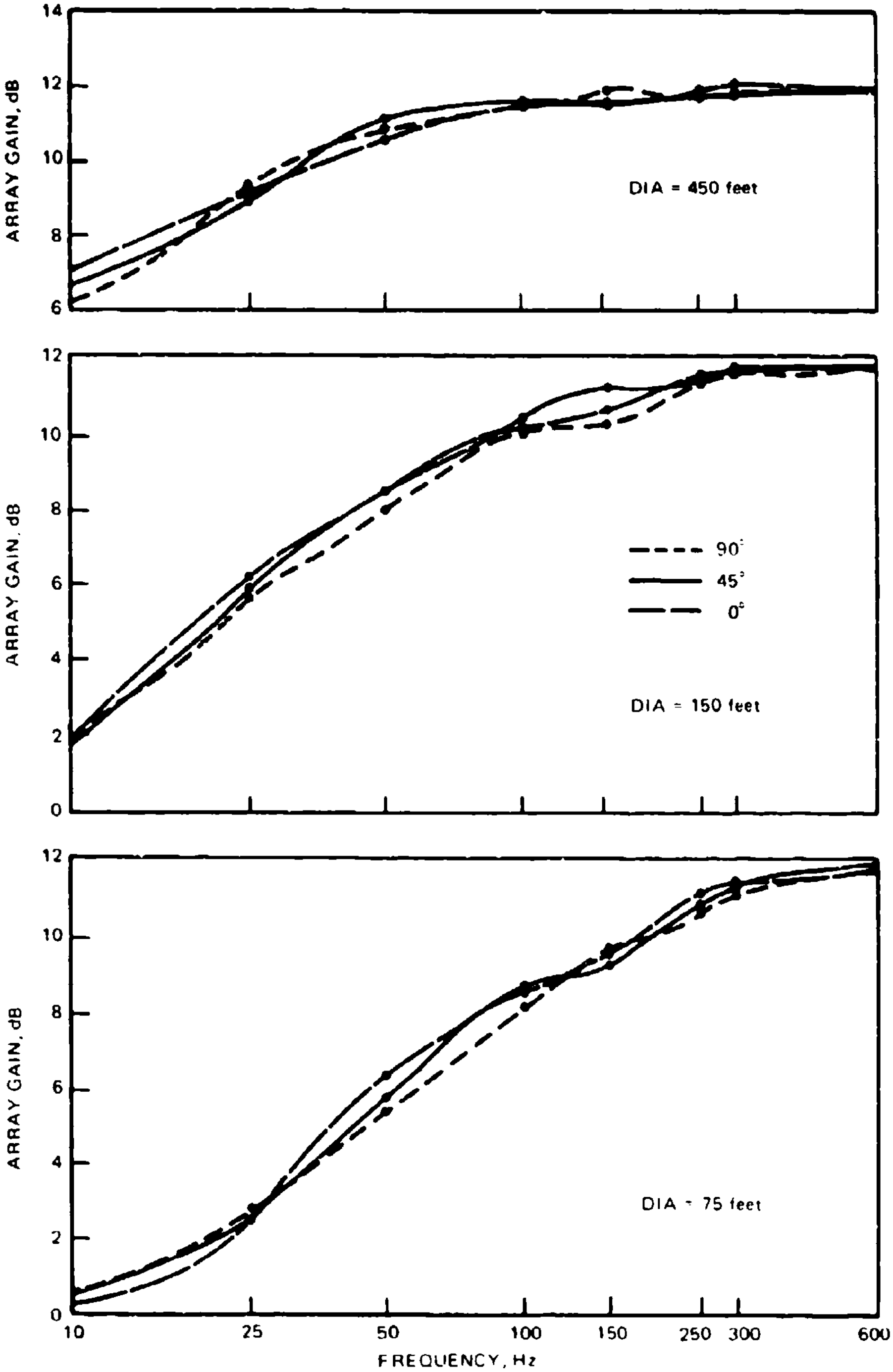

Figure 12. Effects of horizontal steering angle on gain in vertically directive noise.

f_D, Hz	25	50	75	100	125	150	225	300
Dia, feet	900	450	300	225	180	150	100	75

Iso-opt elements

Freq, Hz								
10	11.9	8.9	7.8	7.1	6.7	6.5	6.1	6.0
25	15.5	12.9	11.2	9.9	8.9	8.3	7.3	6.8
50	17.0	15.5	14.2	12.9	11.9	11.2	9.4	8.3
100	17.4	17.1	16.2	15.5	14.8	14.2	12.3	11.2
150	17.7	17.4	17.1	16.4	15.9	15.5	14.2	12.9
250	17.9	17.4	17.4	17.0	17.1	16.7	15.8	14.9
300	17.6	17.7	17.5	17.4	17.1	17.1	16.2	15.5
600	17.7	17.6	17.7	17.7	17.2	17.5	17.3	17.1

Omnidirectional elements

Freq, Hz								
10	9.5	6.6	4.8	3.2	2.2	1.6	0.7	0.4
25	11.6	9.4	9.2	7.8	6.6	5.9	3.8	2.4
50	12.0	12.0	10.9	9.4	9.5	9.2	7.1	5.9
100	11.8	12.1	11.4	11.6	11.1	10.9	9.3	9.2
150	12.0	11.9	12.1	11.8	11.4	11.6	10.9	9.4
250	12.1	11.8	11.8	11.6	12.1	11.7	11.6	11.2
300	11.9	12.0	11.8	11.9	11.7	12.1	11.4	11.6
600	12.0	11.9	12.2	12.0	11.8	11.8	12.0	12.1

Improvement in gain due to iso-opt elements

Freq, Hz								
10	2.4	2.3	3.0	3.9	4.5	4.9	5.4	5.6
25	3.9	3.5	2.0	2.1	2.3	2.4	3.5	4.4
50	5.0	3.5	3.3	3.5	2.4	2.0	2.3	2.4
100	5.6	5.0	4.8	3.9	3.7	3.3	3.0	2.0
150	5.7	5.5	5.0	5.6	4.5	3.9	3.3	3.5
250	5.8	5.6	5.6	5.4	5.0	5.0	4.2	3.7
300	5.7	5.7	5.7	5.5	5.4	5.0	4.8	3.9
600	5.7	5.7	5.5	5.7	5.4	5.7	5.3	5.0

Table 7. Directivity indices (dB) for horizontal disc random arrays in isotropic noise ($N = 16$, $\phi_S = 45°$, $\theta_S = 90°$).

DIRECTIONAL RESPONSE IN HORIZONTAL AND VERTICAL PLANES

RESOLUTION

Bearing resolution will be defined in terms of the beam widths of the arrays. To obtain a general relationship for discrete random disc arrays, the directional responses for solid disc arrays will be derived.

DIRECTIONAL RESPONSE OF A HORIZONTAL SOLID THIN DISC

The normalized directional response of a horizontal thin disc is given by the relationship

$$p = \frac{2J_1(\psi)}{\psi},$$

where

$$\psi = kaV$$

$$(V)^2 = [\sin(\theta_S)\cos(\phi_S) - \sin(\theta)\cos(\phi)]^2 + [\sin(\theta_S)\sin(\phi_S) - \sin(\theta)\sin(\phi)]^2.$$

For the half-power beam width,

$$p = \frac{\sqrt{2}}{2} \Rightarrow kaV = \pm 1.616\,34,$$

so that

$$V = \frac{\pm 1.616\,34}{ka}$$

$$= \frac{\pm 0.5145}{\left(\frac{D}{\lambda}\right)},$$

with the diameter

$$D = 2a.$$

HORIZONTAL BEAM WIDTH OF A SOLID DISC

For the horizontal beam width,

$$\theta = \theta_S = \frac{\pi}{2}$$

$$V = 2\sin\left(\frac{\phi - \phi_S}{2}\right)$$
$$= \frac{\pm 0.5145}{\left(\frac{D}{\lambda}\right)} .$$

In view of the symmetry of the two solutions, the beam width, BW, in degrees, is as follows:

$$BW = \frac{360}{\pi} |\phi - \phi_S|$$
$$= \frac{720}{\pi} \sin^{-1}\left[\frac{0.5145}{2\left(\frac{D}{\lambda}\right)}\right] ,$$

so that

$$BW = 229.18^\circ \sin^{-1}\left[\frac{0.25725}{\left(\frac{D}{\lambda}\right)}\right] .$$

If the disc is sufficiently large,

$$BW \cong \frac{58.96^\circ}{\left(\frac{D}{\lambda}\right)} .$$

This beam width is similar in form to that of an unshaded linear array except that it is about 15% broader because of the inherent spatial shading.

If the disc is steered vertically, the horizontal beam width solution is of nearly identical form.

$$\theta = \theta_S$$
$$V = 2\sin(\theta_S)\sin[(\theta - \theta_S)/2]$$
$$BW = \frac{720}{\pi}\sin^{-1}\left[\frac{1.61634}{2\pi\left(\frac{D}{\lambda}\right)\sin(\theta_S)}\right]$$

and, for large $\frac{D}{\lambda}$,

$$BW \cong \frac{58.96^\circ}{\left(\frac{D}{\lambda}\right)\sin(\theta_S)} .$$

VERTICAL BEAM WIDTH OF A SOLID DISC

Consider the vertical beam width where the disc is steered to the proper azimuth so that

$$\phi = \phi_S \text{ and } V = \pm(\sin\theta - \sin\theta_S).$$

Let us define

$$\alpha_1 = \sin\theta_S + 0.5145/(D/\lambda)$$

and

$$\alpha_2 = \sin\theta_S - 0.5145/(D/\lambda),$$

so that beam width is given by the relationships

$$BW = \sin^{-1}(\alpha_1) - \sin^{-1}(\alpha_2) \text{ (if } \alpha_1 < 1.0)$$

$$BW = \pi - 2\sin^{-1}(\alpha_2) \text{ (if } \alpha_1 > 1.0)$$

and the beam width doubles to its maximum value, BW_{max}, as α_1 passes through unity, where

$$BW_{max} = 4\sin^{-1}\sqrt{0.5145\ (D/\lambda)}$$

and

$$BW_{max} = 4\cos^{-1}\sqrt{\sin\theta_S}.$$

Two simple equations can be obtained if the disc is large. When the steering is relatively horizontal or near edge-fire ($\theta_S \approx \pi/2$), then

$$BW \approx \frac{116.24°}{\sqrt{\frac{D}{\lambda}\sin\theta_S}} \qquad \text{for } \tan\left(\frac{BW}{4}\right)\tan(\theta_S) \gg 1.$$

When the steering of the disc is nearly vertical or near broadside ($\theta_S \approx 0$), then

$$BW \approx \frac{58.957°}{\frac{D}{\lambda}\cos\theta_S} \qquad \text{for } \tan\left(\frac{BW}{4}\right)\tan\theta_S \ll 1.$$

Disc arrays employed with edge-fire operation have been of interest to NOSC for years. Directional responses such as the edge-fire beam width have been reported (ref 6, 12).

HORIZONTAL BEAM WIDTH FOR HORIZONTAL DISC RANDOM ARRAY

In horizontal planes, the directional response of an array of N omnidirectional elements is as follows:

$$p_{HOR} = \frac{1}{N}\sum_{\ell=1}^{N} e^{ik\{x(\ell)[\cos(\phi)-\cos(\phi_S)]+y(\ell)[\sin(\phi)-\sin(\phi_S)]\}},$$

12. Available to qualified requestors.

where $x(\ell)$ and $y(\ell)$ are the coordinates of the ℓth element of the array located in the horizontal ($z = 0$) plane. The responses were computed near the main-response axis. Half-power beam widths were determined by recognizing that

$$DB = 10 \log_{10} (|p_{HOR}|^2)$$
$$\propto (\phi - \phi_S)^2 .$$

We form estimates for ϕ such that

$$\phi_1^0 \cong \phi_S + \frac{BW}{2}$$

and

$$\phi_2^0 \cong \phi_S - \frac{BW}{2} .$$

A very useful estimator is given by the following relationship:

$$\phi_n \cong \phi_n^1 \equiv \phi_S + (\phi_n^0 - \phi_S) \sqrt{DB(\phi_n^0, \phi_S)/(-3.0103)} . \qquad (n = 1 \text{ or } 2)$$

Multiple iterations may be required for sufficient precision. The beam width is given by the relationship

$$BW = \phi_1 - \phi_2 .$$

The resultant beam widths for the 16-element random arrays are presented in table 8. The beam widths are also given for the random arrays with cardioid and iso-opt elements. Even though the beam widths vary appreciably, the product of beam width and frequency, as theory predicts, is nearly constant except at low frequencies. Using the design parameters for the arrays and the solid-disc beam width equation leads to the theoretical relation for a solid disc,

$$BW \left(\frac{f}{f_D} \right) = 13.06° ,$$

which agrees well with the reported random-array results.

VERTICAL BEAM WIDTH FOR HORIZONTAL DISC RANDOM ARRAY

In vertical planes, the directional response of an array of N omnidirectional elements is given by the relationship

$$p_{VERT} = \frac{1}{N} \sum_{\ell=1}^{N} e^{ik[x(\ell) \cos(\phi_S) + y(\ell) \sin(\phi_S)] \; [\sin(\theta) - \sin(\theta_S)]} .$$

Half-power beam widths were determined, based upon the approximation that

$$10 \log_{10} (|p_{HOR}|^2) \cong \beta (\theta - \theta_S)^4 ,$$

where β is a constant to be determined

Beam width, °

Freq, Hz	f_D = 50			f_D = 150			f_D = 300		
	Omni	Cardioid	Iso-opt	Omni	Cardioid	Iso-opt	Omni	Cardioid	Iso-opt
10	65.7	59.1	56.3	239.	111.6	94.2	360.	125.5	102.0
50	12.8	12.8	12.7	38.8	37.4	36.7	79.8	68.4	64.2
100	6.41	6.40	6.40	19.3	19.1	19.0	38.8	37.3	36.7
150	4.27	4.27	4.27	12.8	12.8	12.7	25.7	25.3	25.1
250	2.56	2.56	2.56	7.69	7.68	7.67	15.4	15.3	15.3
300	2.14	2.14	2.14	6.41	6.40	6.40	12.8	12.7	12.7
600	1.07	1.07	1.07	3.20	3.20	3.20	6.41	6.40	6.40

$(f/f_D) \times$ beam width, °

Freq, Hz	f_D = 50			f_D = 150			f_D = 300		
	Omni	Cardioid	Iso-opt	Omni	Cardioid	Iso-opt	Omni	Cardioid	Iso-opt
10	13.1	11.8	11.3	15.9	7.44	6.28	X	4.18	3.40
50	12.8	12.8	12.7	12.9	12.5	12.2	13.3	11.4	10.7
100	12.8	12.8	12.8	12.9	12.7	12.7	12.9	12.4	12.2
150	12.8	12.8	12.8	12.8	12.8	12.7	12.9	12.7	12.5
250	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.7
300	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8
600	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8

Table 8. Horizontal beam widths for horizontal disc random arrays ($N = 16$, $\phi_S = 45°$, $\theta_S = 90°$).

An iterative method, similar to the horizontal beam width technique, was used for the vertical beam width calculation.

The resultant beam widths for 16-element random arrays are presented in table 9. The product of the beam width and the square root of the relative frequency is nearly constant, in agreement with the theory for the solid disc, which indicates the following:

$$BW\sqrt{f/f_D} = 54.71°.$$

Some of the beam widths are sufficiently narrow that the signal loss will be significant for many target ranges. The vertical beam widths for the arrays with high design frequency are broader than those of the arrays with lower design frequencies. Thus the signal loss will be more pronounced for larger arrays with lower design frequencies. These signal loss effects can be eliminated by steering the arrays vertically to accommodate the signal arrivals of interest. The thin disc will have up-down symmetry, so that vertical steering produces downward and upward beams that intercept both kinds of signal (and noise) arrivals.

Beam width, °

Freq.	$f_D = 50$			$f_D = 150$			$f_D = 300$		
Hz	Omni	Cardioid	Iso-opt	Omni	Cardioid	Iso-opt	Omni	Cardioid	Iso-opt
10	129.0	102.2	91.2	X	125.1	102.7	X	129.3	104.2
50	55.2	53.0	51.9	97.6	85.6	79.8	143.0	107.5	94.4
100	38.9	38.1	37.7	68.0	63.9	61.8	97.6	85.6	79.8
150	31.7	31.3	31.1	55.2	53.0	51.9	78.9	72.5	69.3
250	24.5	24.3	24.2	42.6	41.6	41.1	60.6	57.7	56.3
300	22.4	22.2	22.1	38.9	38.1	37.7	55.2	53.0	51.9
600	15.8	15.8	15.7	27.4	27.1	27.0	38.9	38.1	38.1

$(f/f_D) \times$ beam width, °

Freq.	$f_D = 50$			$f_D = 150$			$f_D = 300$		
Hz	Omni	Cardioid	Iso-opt	Omni	Cardioid	Iso-opt	Omni	Cardioid	Iso-opt
10	57.7	45.7	40.8	X	32.3	45.9	X	23.6	19.0
50	55.2	53.0	51.9	56.4	49.4	46.1	58.4	43.9	38.5
100	54.9	53.9	53.3	55.5	52.2	50.5	56.4	49.4	46.1
150	54.9	54.2	53.9	55.2	53.0	51.9	55.8	51.3	49.0
250	54.8	54.3	54.1	55.0	53.7	53.1	55.3	52.7	51.4
300	54.8	54.4	54.1	54.9	53.9	53.3	55.2	53.0	51.9
600	54.7	54.7	54.4	54.8	54.2	54.0	54.8	53.9	53.9

Table 9. Vertical beam widths for horizontal disc random arrays ($N = 16$, $\phi_S = 45°$, $\theta_S = 90°$).

HORIZONTAL DIRECTIONAL RESPONSE FOR HORIZONTAL DISC RANDOM ARRAY

The directional response in a horizontal plane, p_{HOR}, can be calculated for all azimuths. Several sets of these responses are presented here to illustrate the general character of the patterns, including the sidelobe levels.

The change of directional response with frequency is shown in figure 13. Since all arrays are proportional in dimensions, patterns for all arrays are the same for equal relative frequencies, f/f_D. Results are illustrated for three values of relative frequencies: 0.5, 1.0, and 2.0. The beam width is about inversely proportional to frequency, as discussed above. The sidelobe levels are relatively poor for arrays with omnidirectional elements, but the results are typical. Random arrays generally have mean sidelobe levels of $-10 \log_{10} (N)$, which amounts to −12 dB for arrays of 16 elements. The backward responses are improved substantially by the use of cardioid and iso-opt directional hydrophone elements, but the forward responses are nearly unaffected by change of element directionality. In general, the directional discrimination of these unshaded arrays is much superior to that of a single hydrophone but sufficiently inadequate that false-target problems could result from off-axis sources.

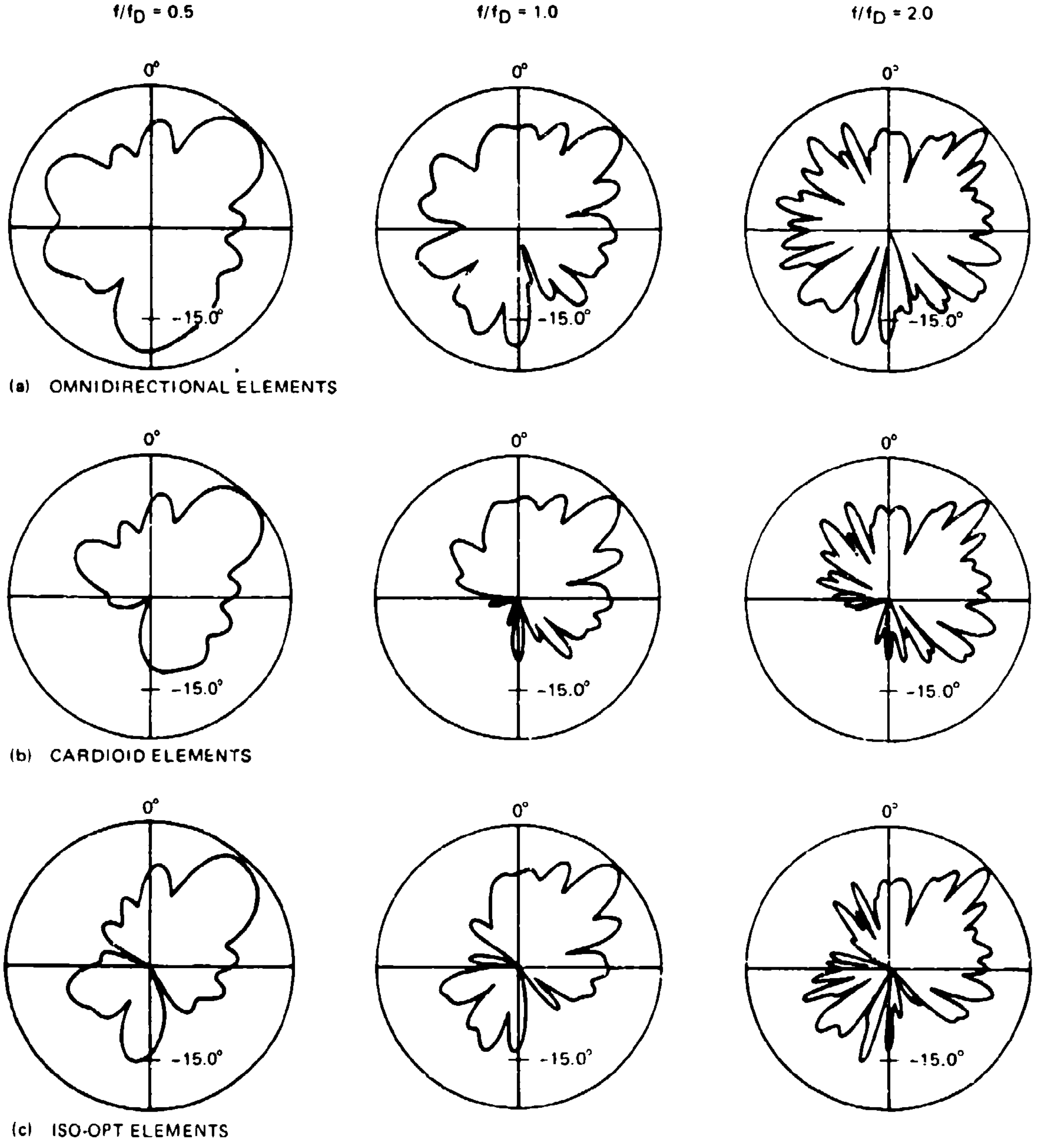

Figure 13. Horizontal directional response of 16-element horizontal disc random arrays.

The 50 Hz directional responses of three arrays with design frequencies of 50, 150, and 300 Hz are shown in figure 14. Many of the observations are similar to those concerning figure 13. The larger arrays with lower design frequencies (and higher noise gain) exhibit the greater directional discrimination.

Figures 15 and 16 show the 150 Hz and 300 Hz directional responses, respectively.

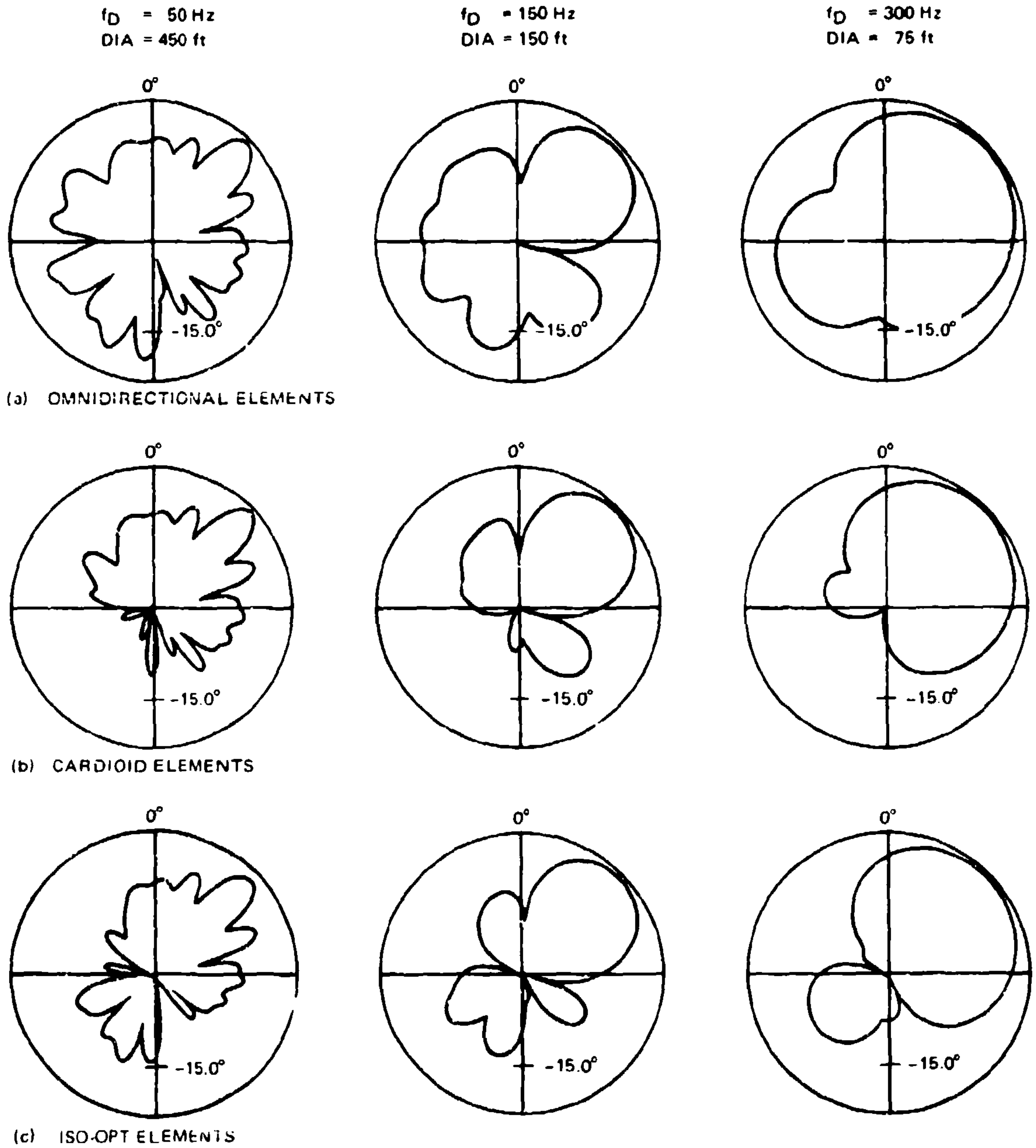

Figure 14. Horizontal directional response of horizontal disc random arrays, f = 50 Hz.

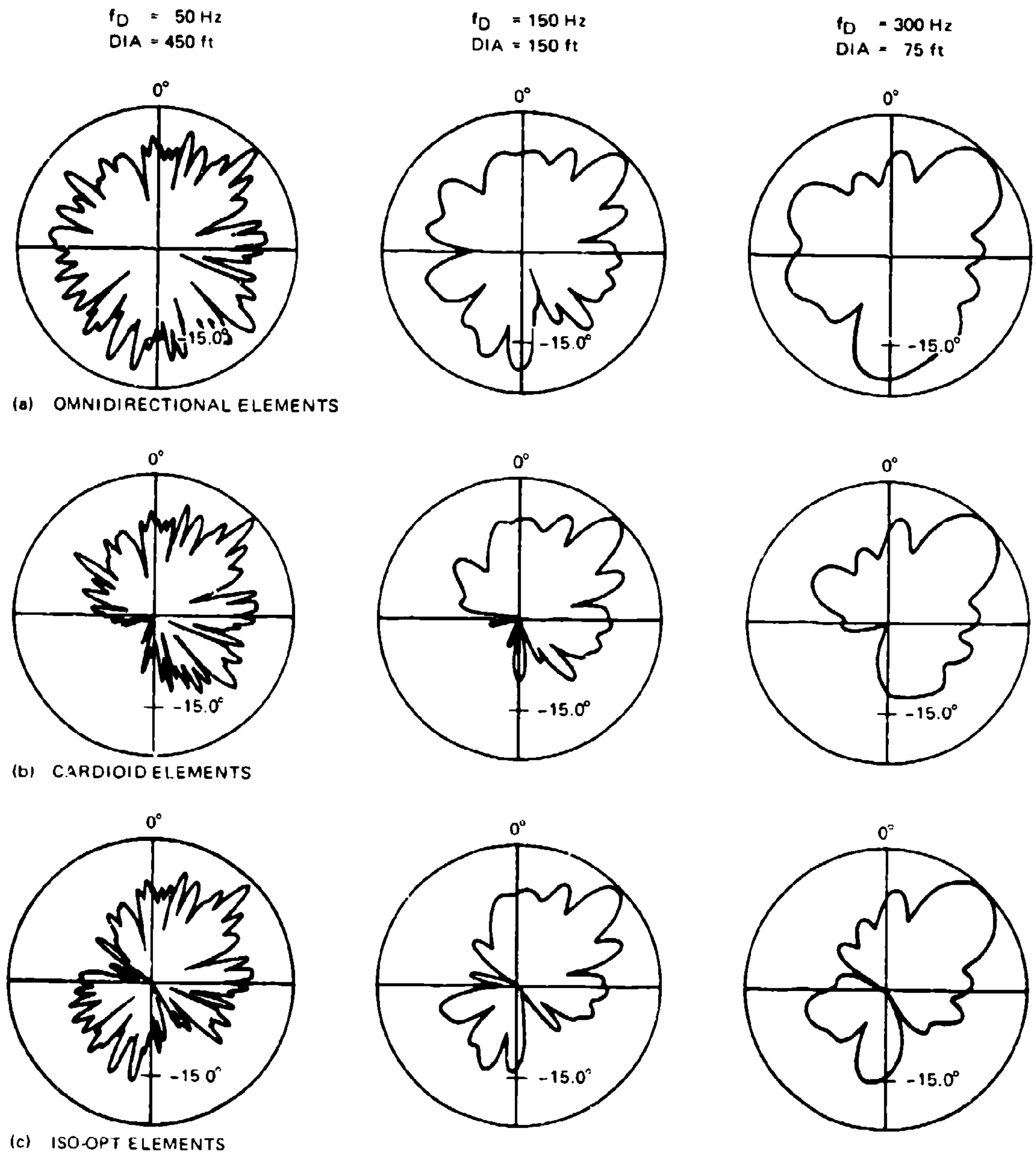

Figure 15. Horizontal directional response of horizontal disc random arrays, f = 150 Hz.

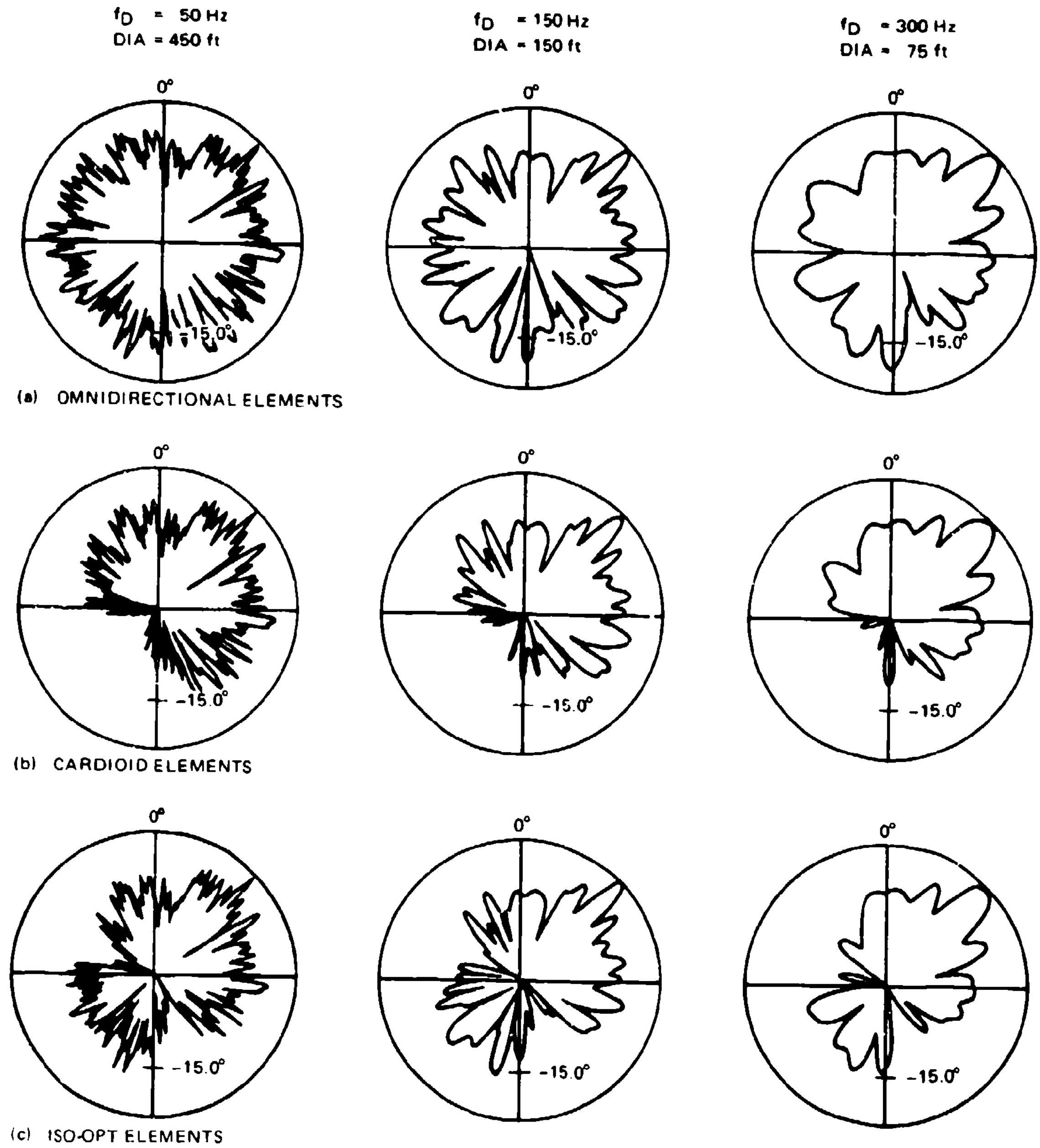

Figure 16. Horizontal directional response of horizontal disc random arrays, f = 300 Hz.

CONCLUSIONS

One class of disc array with 16 randomly located elements has been investigated. In both vertically directive and isotropic noise fields, array noise gain results are similar. There is a significant (3 to 6 dB) improvement of array gain due to the use of limaçon directional hydrophones in the arrays operated at frequencies for which the mean interelement spacing exceeds one wavelength or is less than 0.1 λ. Noise gain is presented for a wide range of frequencies so that the undersampled and oversampled cases are documented well.

It was shown that arrays with directional elements have nearly maximum gain with average interelement spacing of about 1 wavelength if the design frequency (f_D) is 300 Hz, but 5-wavelength spacing is required if f_D is 25 Hz.

The improvement due to the use of an array of 16 iso-opt hypercardioid elements compared to the use of a single directional element alone was evaluated. The improvement is a few dB less than the gain of the array that uses omnidirectional elements.

The horizontal disc array has nearly constant gain with azimuth, even though there is a substantial asymmetry of element locations. This observation is useful in both detection (gain) and localization (resolution) considerations.

The directional responses of random disc arrays were considered in several aspects. The beam widths in both horizontal and vertical planes were shown to approximate those of a solid horizontal disc. The vertical beam width is much wider than the horizontal beam width for the edge-fire disc. Large disc arrays such as those with a design frequency of 50 Hz will have a significant negative signal gain for signals arriving at angles typical of many realistic environmental conditions. Their gain can be boosted by the use of vertical steering.

The directivity patterns have relatively high average sidelobe levels of about −12 dB, but this is to be expected for random arrays with 16 elements.

RECOMMENDATIONS

Consider the horizontal disc array, including the random form discussed, for possible applications. Investigate lobe suppression with regular finite disc arrays. Investigate ocean engineering of both regular and random discs.

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9. Methods of Theoretical Physics, by PM Morse and H Feshbach, vol I, p 658, McGraw-Hill Book Co, NY, 1953.

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12. Available to qualified requestors.

APPENDIX A: ARRAY NOISE GAINS FOR HORIZONTAL DISC RANDOM ARRAYS IN VERTICALLY DIRECTIVE NOISE AND ISOTROPIC NOISE ($N = 16$, $\phi_S = 45°$, $\theta_S = 90°$)

F	FD	F/FD	RAD	ARRAY GAINS IN VERTICALLY DIRECTIVE NOISE			DIRECTIVITY INDICES IN ISO-TROPIC NOISE		
				DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
10.0	25.00	.4000	450.0	10.52	8.68	1.84	11.90	9.48	2.42
25.0	25.00	1.0000	450.0	13.46	11.20	2.26	15.47	11.59	3.88
50.0	25.00	2.0000	450.0	15.03	11.58	3.45	17.03	12.01	5.02
100.0	25.00	4.0000	450.0	16.03	11.62	4.41	17.42	11.82	5.60
150.0	25.00	6.0000	450.0	16.67	11.66	5.01	17.67	11.96	5.71
250.0	25.00	10.0000	450.0	17.76	12.07	5.69	17.90	12.06	5.84
300.0	25.00	12.0000	450.0	17.85	11.94	5.91	17.64	11.93	5.71
600.0	25.00	24.0000	450.0	18.35	12.10	6.25	17.72	12.03	5.69

F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
10.0	50.00	.2000	225.0	8.02	6.52	1.50	8.94	6.61	2.33
25.0	50.00	.5000	225.0	11.40	8.94	2.46	12.91	9.44	3.47
50.0	50.00	1.0000	225.0	13.46	11.20	2.26	15.47	11.59	3.88
100.0	50.00	2.0000	225.0	15.56	11.74	3.82	17.05	12.05	5.00
150.0	50.00	3.0000	225.0	16.39	11.64	4.75	17.40	11.87	5.53
250.0	50.00	5.0000	225.0	17.18	11.76	5.42	17.35	11.81	5.54
300.0	50.00	6.0000	225.0	17.80	11.90	5.90	17.70	11.98	5.72
600.0	50.00	12.0000	225.0	18.24	11.99	6.25	17.63	11.93	5.70

F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
10.0	75.00	.1333	150.0	6.94	5.02	1.92	7.75	4.80	2.95
25.0	75.00	.3333	150.0	9.95	8.65	1.30	11.16	9.18	1.98
50.0	75.00	.6667	150.0	12.42	10.26	2.16	14.23	10.87	3.36
100.0	75.00	1.3333	150.0	14.54	10.73	3.81	16.21	11.38	4.83
150.0	75.00	2.0000	150.0	15.83	11.78	4.05	17.05	12.05	5.00
250.0	75.00	3.3333	150.0	17.02	11.69	5.33	17.35	11.80	5.55
300.0	75.00	4.0000	150.0	17.54	11.79	5.75	17.45	11.84	5.61
600.0	75.00	8.0000	150.0	18.36	12.22	6.14	17.70	12.15	5.55

F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
10.0	100.00	.1000	112.5	6.32	3.51	2.81	7.11	3.21	3.90
25.0	100.00	.2500	112.5	8.89	7.68	1.21	9.88	7.77	2.11
50.0	100.00	.5000	112.5	11.40	8.94	2.46	12.91	9.44	3.47
100.0	100.00	1.0000	112.5	13.98	11.28	2.70	15.53	11.60	3.93
150.0	100.00	1.5000	112.5	15.21	11.66	3.55	16.43	11.82	4.61
250.0	100.00	2.5000	112.5	16.70	11.49	5.21	17.03	11.60	5.43
300.0	100.00	3.0000	112.5	17.52	11.87	5.65	17.42	11.87	5.55
600.0	100.00	6.0000	112.5	18.32	11.99	6.33	17.70	11.99	5.71

				ARRAY GAINS IN VERTICALLY DIRECTIVE NOISE			DIRECTIVITY INDICES IN ISO-TROPIC NOISE		
F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
10.0	125.00	.0800	90.0	5.92	2.43	3.49	6.72	2.19	4.53
25.0	125.00	.2000	90.0	8.02	6.52	1.50	8.94	6.61	2.33
50.0	125.00	.4000	90.0	10.52	8.68	1.84	11.90	9.48	2.42
100.0	125.00	.8000	90.0	13.33	10.57	2.76	14.82	11.09	3.73
150.0	125.00	1.2000	90.0	14.56	10.87	3.69	15.89	11.39	4.50
250.0	125.00	2.0000	90.0	16.67	11.89	4.78	17.07	12.06	5.01
300.0	125.00	2.4000	90.0	17.22	11.66	5.56	17.12	11.74	5.38
600.0	125.00	4.8000	90.0	17.84	11.78	6.06	17.20	11.80	5.40

F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
10.0	150.00	.0667	75.0	5.65	1.75	3.90	6.48	1.57	4.91
25.0	150.00	.1667	75.0	7.47	5.88	1.59	8.34	5.88	2.46
50.0	150.00	.3333	75.0	9.95	8.65	1.30	11.16	9.18	1.98
100.0	150.00	.6667	75.0	12.88	10.43	2.45	14.24	10.89	3.35
150.0	150.00	1.0000	75.0	14.32	11.33	2.99	15.53	11.60	3.93
250.0	150.00	1.6667	75.0	16.38	11.55	4.83	16.69	11.68	5.01
300.0	150.00	2.0000	75.0	17.12	11.93	5.19	17.07	12.06	5.01
600.0	150.00	4.0000	75.0	18.11	11.83	6.28	17.45	11.84	5.61

F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
10.0	225.00	.0444	50.0	5.26	.81	4.45	6.12	.72	5.40
25.0	225.00	.1111	50.0	6.54	4.08	2.46	7.33	3.78	3.55
50.0	225.00	.2222	50.0	8.41	7.02	1.39	9.36	7.11	2.25
100.0	225.00	.4444	50.0	11.25	8.74	2.51	12.31	9.32	2.99
150.0	225.00	.6667	50.0	13.22	10.53	2.69	14.24	10.89	3.35
250.0	225.00	1.1111	50.0	15.39	11.29	4.10	15.78	11.60	4.18
300.0	225.00	1.3333	50.0	16.41	11.45	4.96	16.23	11.39	4.84
600.0	225.00	2.6667	50.0	17.98	12.07	5.91	17.27	11.97	5.30

F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
10.0	300.00	.0333	37.5	5.11	.46	4.65	5.99	.41	5.58
25.0	300.00	.0833	37.5	5.99	2.61	3.38	6.79	2.36	4.43
50.0	300.00	.1667	37.5	7.47	5.88	1.59	8.34	5.88	2.46
100.0	300.00	.3333	37.5	10.30	8.84	1.46	11.15	9.19	1.96
150.0	300.00	.5000	37.5	12.11	9.25	2.86	12.91	9.44	3.47
250.0	300.00	.8333	37.5	14.67	10.97	3.70	14.93	11.17	3.76
300.0	300.00	1.0000	37.5	15.71	11.46	4.25	15.54	11.61	3.93
600.0	300.00	2.0000	37.5	17.78	11.98	5.80	17.07	12.06	5.01

F	FD	F/FD	RAD	ARRAY GAINS IN VERTICALLY DIRECTIVE NOISE DB ISO-OPT	DB OMNI	DB DIFF	DIRECTIVITY INDICES IN ISO-TROPIC NOISE DB ISO-OPT	DB OMNI	DB DIFF
10.0	25.00	.4000	450.0	10.52	8.68	1.84	11.90	9.48	2.42
10.0	50.00	.2000	225.0	8.02	6.52	1.50	8.94	6.61	2.33
10.0	75.00	.1333	150.0	6.94	5.02	1.92	7.75	4.80	2.95
10.0	100.00	.1000	112.5	6.32	3.51	2.81	7.11	3.21	3.90
10.0	125.00	.0800	90.0	5.92	2.43	3.49	6.72	2.19	4.53
10.0	150.00	.0667	75.0	5.65	1.75	3.90	6.48	1.57	4.91
10.0	225.00	.0444	50.0	5.26	.81	4.45	6.12	.72	5.40
10.0	300.00	.0333	37.5	5.11	.46	4.65	5.99	.41	5.58
F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
25.0	25.00	1.0000	450.0	13.46	11.20	2.26	15.47	11.59	3.88
25.0	50.00	.5000	225.0	11.40	8.94	2.46	12.91	9.44	3.47
25.0	75.00	.3333	150.0	9.95	8.65	1.30	11.16	9.18	1.98
25.0	100.00	.2500	112.5	8.89	7.68	1.21	9.88	7.77	2.11
25.0	125.00	.2000	90.0	8.02	6.52	1.50	8.94	6.61	2.33
25.0	150.00	.1667	75.0	7.47	5.88	1.59	8.34	5.88	2.46
25.0	225.00	.1111	50.0	6.54	4.08	2.46	7.33	3.78	3.55
25.0	300.00	.0833	37.5	5.99	2.61	3.38	6.79	2.36	4.43
F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
50.0	25.00	2.0000	450.0	15.03	11.58	3.45	17.03	12.01	5.02
50.0	50.00	1.0000	225.0	13.46	11.20	2.26	15.47	11.59	3.88
50.0	75.00	.6667	150.0	12.42	10.26	2.16	14.23	10.87	3.36
50.0	100.00	.5000	112.5	11.40	8.94	2.46	12.91	9.44	3.47
50.0	125.00	.4000	90.0	10.52	8.68	1.84	11.90	9.48	2.42
50.0	150.00	.3333	75.0	9.95	8.65	1.30	11.16	9.18	1.98
50.0	225.00	.2222	50.0	8.41	7.02	1.39	9.36	7.11	2.25
50.0	300.00	.1667	37.5	7.47	5.88	1.59	8.34	5.88	2.46
F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
100.0	25.00	4.0000	450.0	16.03	11.62	4.41	17.42	11.82	5.60
100.0	50.00	2.0000	225.0	15.56	11.74	3.82	17.05	12.05	5.00
100.0	75.00	1.3333	150.0	14.54	10.73	3.81	16.21	11.38	4.83
100.0	100.00	1.0000	112.5	13.98	11.28	2.70	15.53	11.60	3.93
100.0	125.00	.8000	90.0	13.33	10.57	2.76	14.82	11.09	3.73
100.0	150.00	.6667	75.0	12.88	10.43	2.45	14.24	10.89	3.35
100.0	225.00	.4444	50.0	11.25	8.74	2.51	12.31	9.32	2.99
100.0	300.00	.3333	37.5	10.30	8.84	1.46	11.15	9.19	1.96

				ARRAY GAINS IN VERTICALLY DIRECTIVE NOISE			DIRECTIVITY INDICES IN ISO-TROPIC NOISE		
F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
150.0	25.00	6.0000	450.0	16.67	11.66	5.01	17.67	11.96	5.71
150.0	50.00	3.0000	225.0	16.39	11.64	4.75	17.40	11.87	5.53
150.0	75.00	2.0000	150.0	15.83	11.78	4.05	17.05	12.05	5.00
150.0	100.00	1.5000	112.5	15.21	11.66	3.55	16.43	11.82	4.61
150.0	125.00	1.2000	90.0	14.56	10.67	3.69	15.89	11.39	4.50
150.0	150.00	1.0000	75.0	14.32	11.33	2.99	15.53	11.60	3.93
150.0	225.00	.6667	50.0	13.22	10.53	2.69	14.24	10.89	3.35
150.0	300.00	.5000	37.5	12.11	9.25	2.86	12.91	9.44	3.47

F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
250.0	25.00	10.0000	450.0	17.76	12.07	5.69	17.90	12.06	5.84
250.0	50.00	5.0000	225.0	17.18	11.76	5.42	17.35	11.81	5.54
250.0	75.00	3.3333	150.0	17.02	11.69	5.33	17.35	11.80	5.55
250.0	100.00	2.5000	112.5	16.70	11.49	5.21	17.03	11.60	5.43
250.0	125.00	2.0000	90.0	16.67	11.89	4.78	17.07	12.06	5.01
250.0	150.00	1.6667	75.0	16.38	11.55	4.83	16.69	11.68	5.01
250.0	225.00	1.1111	50.0	15.39	11.29	4.10	15.78	11.60	4.18
250.0	300.00	.8333	37.5	14.67	10.97	3.70	14.93	11.17	3.76

F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
300.0	25.00	12.0000	450.0	17.85	11.94	5.91	17.64	11.93	5.71
300.0	50.00	6.0000	225.0	17.80	11.90	5.90	17.70	11.98	5.72
300.0	75.00	4.0000	150.0	17.54	11.79	5.75	17.45	11.84	5.61
300.0	100.00	3.0000	112.5	17.52	11.87	5.65	17.42	11.87	5.55
300.0	125.00	2.4000	90.0	17.22	11.66	5.56	17.12	11.74	5.38
300.0	150.00	2.0000	75.0	17.12	11.93	5.19	17.07	12.06	5.01
300.0	225.00	1.3333	50.0	16.41	11.45	4.96	16.23	11.39	4.84
300.0	300.00	1.0000	37.5	15.71	11.46	4.25	15.54	11.61	3.93

F	FD	F/FD	RAD	DB ISO-OPT	DB OMNI	DB DIFF	DB ISO-OPT	DB OMNI	DB DIFF
600.0	25.00	24.0000	450.0	18.35	12.10	6.25	17.72	12.03	5.69
600.0	50.00	12.0000	225.0	18.24	11.99	6.25	17.63	11.93	5.70
600.0	75.00	8.0000	150.0	18.36	12.22	6.14	17.70	12.15	5.55
600.0	100.00	6.0000	112.5	18.32	11.99	6.33	17.70	11.99	5.71
600.0	125.00	4.8000	90.0	17.84	11.78	6.06	17.20	11.80	5.40
600.0	150.00	4.0000	75.0	18.11	11.83	6.28	17.45	11.84	5.61
600.0	225.00	2.6667	50.0	17.98	12.07	5.91	17.27	11.97	5.30
600.0	300.00	2.0000	37.5	17.78	11.98	5.80	17.07	12.06	5.01

APPENDIX B: PROGRAM FOR GENERATION OF COORDINATES OF RANDOM-ARRAY ELEMENTS

```
C               COMPUTE ELEMENT LOCATIONS FOR RANDOM ARRAYS BY RL HESSER
C              *** THIS PGM ONLY FOR CIRCULAR ARRAY***
C              *** IN XY PLANE, IE, INCOMPLETE.     ***
C
               DIMENSION X(100),Y(100),Z(100)
C
               READ(5,1001) ICS,IE,EMSD,RAD,IG,RX
               WRITE(6,1008)ICS,IE,EMSD,RAD,IG,RX
               WRITE (6,1004)
               WRITE (6,1002)
               DO 10 I=1,100
               X(I)=0.
               Y(I)=0.
               Z(I)=0.
      10       CONTINUE
C
C              ICS=COORDINATE SYSTEM
C                   X=1
C                   Y=2
C                   Z=3
C                  XY=4
C                  XZ=5
C                  YZ=6
C                 XYZ=7
C                  IE=NO OF ELEMENTS
C
C               EMSD=MINIMUM ELEMENT SEPARATION DISTANCE
C
C                 RAD=RADIUS OF CIRCLE,SIDE OF SQUARE,SHORTEST
C                     SIDE OF RECTANGLE,MINOR AXIS OF ELLIPSE
C
C                  IG=GEOMETRY
C
C                  CIRCLE   =1
C                  SQUARE   =2
C                  RECTANGLE=3
C                  ELLIPSE  =4
C
                 IF(IG.GT.2)READ(5,1001)IRAD2
C
C                  IRAD2=LONGEST SIDE OF RECTANGLE OR MAJOR AXIS
C                        OF ELLIPSE
               RZ  =FLOAT(ICS+IE+IG)*((EMSD/RAD)+.5)+RX
               EMSD=EMSD*1.4142135
               DO 100 I=1,IE
               IF ((ICS .NE. 4) .OR. (IG .NE. 1)) GO TO 100
      50       CALL SS1(RZ)
               X(I) = (RZ + RZ - 1.0) * RAD
               CALL SS1(RZ)
               Y(I) = (RZ + RZ -1.0) * RAD
               IF (EMSD .EQ. 0.0) GO TO 75
               IF (I .GT. 1) CALL CSD(I,X,Y,Z,EMSD,J)
C     75       IF (I .GT. 1) CALL CMD(I,X,Y,Z,RAD,R,J)
C              THE SUBROUTINE WAS CHANGED ALSO.
      75       CALL CMD(X(I),Y(I),Z(I),RAD,R,J)
               IF (J .GT. 0) GO TO 50
```

```
            WRITE (6,1003) I,X(I), Y(I), Z(I), R, RAD
CC          WRITE(10,1003) I,X(I),Y(I),Z(I)
  100       CONTINUE
            WRITE (10,1005) (X(L), L = 17,32)
            WRITE (10,1006) (Y(L), L = 17,32)
            WRITE (10,1007) (Z(L), L = 17,32)
 1001 FORMAT ( )
 1002 FORMAT (21X,'L',7X,'X(L)',7X,'Y(L)',2X,'Z(L)',5X,'R',7X,'RAD')
 1003 FORMAT (18X, I4, 2(2X, F9.2),F6.2,2F9.2)
 1004 FORMAT (//)
 1005 FORMAT ('X= ', 4F10.4,3(/'*   ',4F10.4))
 1006 FORMAT ('Y= ', 4F10.4,3(/'*   ',4F10.4) )
 1007 FORMAT ('Z= ', 4F10.4,3(/'*   ',4F10.4))
 1008 FORMAT (' ICS=', I2, 2X, 'IE=', I2, 2X, 'EMSD=', F4.1,
     & 2X,'RAD=', F7.2, 2X, 'IG=', I2, 2X, 'RX=',F5.1 )
      END
```

```
      SUBROUTINE CMD(X,Y,Z,RAD,R,J)
      J=-1
      R=SQRT(X**2+Y**2)
      IF(R.GT.RAD) J=1
      RETURN
      END
```

```
      SUBROUTINE CSD(I,X,Y,Z,EMSD,J)
      DIMENSION X(100),Y(100),Z(100)
      J=-1
      K=I-1
      DO 100 L=1,K
      XI=X(I)-X(L)
      YI=Y(I)-Y(L)
      ZI=SQRT(XI**2+YI**2)
      IF(ZI.LE.EMSD) GO TO 200
  100 CONTINUE
      RETURN
  200 J=1
      RETURN
      END
```

INPUT PARAMETERS TO APPENDIX B PROGRAM USED TO OBTAIN RANDOM ARRAY LOCATIONS

ICS	4	Horizontal planar array
IE	32	Elements
EMSD	0.0	Minimum separation
RAD	1.0	Radius
IG	1	Circular array
RX	10	Random number parameter